

REPO RATES AND THE COLLATERAL SPREAD PUZZLE: THEORY AND EVIDENCE *

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Abstract

Repo Rates and the Collateral Spread Puzzle: Theory and Evidence

The collateral spread is the unsecured rate less the repo rate. The puzzle is that this is frequently negative. To understand this, we develop a theory where repos are motivated by the need to raise liquidity. The unsecured and the security cash markets are alternative sources. Unsecured borrowing constraints generate a constrained-arbitrage relation between the repo rate, the (adjusted) expected rate of return of the underlying security, and the unsecured rate. Collateral spreads can turn negative if borrowing constraints tighten, unsecured rates spike down, or from depressed and illiquid securities markets. Collateral spreads increase in haircuts and decrease in the volatility, illiquidity, and expected rate of return of the underlying collateral. Empirical tests using comprehensive data from Eurex Repo are supportive. We use the theory to provide a narrative of the evolution of collateral spreads from before to after the financial crisis.

Keywords: collateral spread, liquidity, unsecured rate, repo rate, general collateral, Eurex Repo

JEL: G01, G12, G21

1 Introduction

Repurchase agreements (repos) are often characterized as being, in effect, a type of collateralized loan (Duffie, 1996). Repo rates would therefore be expected to be lower than unsecured rates, which indeed they typically are. However, a puzzling feature of the market for liquidity is that repo rates shoot above unsecured rates from time to time, sometimes even for extended periods. This is illustrated in Figures 1a and 1b. The figures graph what we call the *collateral spread*, defined as the difference between the unsecured rate and a repo rate. Thus, a negative value of the collateral spread means that the repo rate is higher than the unsecured rate. The figure shows that during the early stage of the financial crisis, repo rates in the euro area were several basis points above unsecured rates for prolonged periods of time.¹ We can also see that repo rates occasionally went above unsecured rates also prior to the crisis and have continued to do so later on. Over the sample periods in Figures 1a and b, the collateral spread is negative approximately 25% of the time. Negative collateral spreads can also be seen in US data (Bartolini, Hilton, Sundaresan, and Tonetti, 2011).²

[insert Figure 1 about here]

A negative collateral spread is puzzling because repos are typically viewed as reducing credit risk relative to unsecured borrowing. The long periods of negative collateral spreads shown in Figure 1 are all the more surprising because the repo rates that we have used

¹Figure 1 shows overnight collateral spreads. The unsecured rate is the Eonia, which is a volume-weighted average of overnight transactions (for euros) by reporting (European) banks. Eonia is an acronym for Euro Overnight Index Average. See <http://www.euribor-rates.eu/eonia.asp>. The repo rates are volume-weighted averages of all overnight transactions in Eurex Repo's two general collateral contracts, General Collateral Pooling ECB basket and General Collateral Pooling ECB Extended basket. These are among the most active repo contracts in the euro market and are described more fully in Section 4. See also Mancini, Ranaldo, and Wrampelmeyer (2016) for evidence on volume in these contracts. The collateral spread is negative 22.9% and 28.7% of the time in Figures 1a and b, respectively.

²The occurrence of repo rates larger than unsecured rates can be seen in the summary statistics presented by Bartolini et al. (2011), but is not the focus of their study. In contrast to what we do in this paper, they define the collateral spread as the repo rate less the unsecured rate.

are from a central counterparty (CCP), which should eliminate credit risk. In this paper, we seek to understand the puzzle of negative collateral spreads and, more generally, shed light on the behavior of repo versus unsecured rates.

Improving our understanding of repo and unsecured rates is important for a number of reasons. First, in many countries and currency areas, these rates are important vehicles for the implementation of monetary policy. Second, it has been argued that the market for liquidity is central to the financial system (see, among others, Bindseil, Nyborg, and Strebulaev, 2009; Afonso, Kovner, and Schoar, 2011; Fecht, Nyborg, and Rocholl, 2011; Gorton and Metrick, 2011). It may interact with securities markets (Brunnermeier and Pedersen, 2009), and frictions in the interbank market spill over to the broader financial markets (Nyborg and Östberg, 2014). Third, interbank rates are used as reference rates in mortgages, various credit agreements, and in derivatives markets. The significance of these rates to society as a whole is emphasized by the public and regulatory outrage at the manipulation of Libor by a number of banks.³ In the wake of the Libor scandal, the Financial Stability Board (FSB) in Basel and others have called for Libor to be replaced by a repo rate benchmark.⁴ This points to the importance of rates set in the market for liquidity, especially in the post-crisis landscape, and Figures 1a and 1b underscore that we have much to learn about them. In this paper, we first study the relation between repo rates and unsecured rates theoretically and then proceed to empirically test some of the implications of our theory.

As discussed by Duffie (1996), repos are often used as vehicles by cash takers to finance the purchase of the underlying collateral. They can also be driven by cash providers' objectives of obtaining particular securities. Duffie's focus is on such *special* repos.

In contrast, the theory we develop is driven by the use of repos to obtain liquidity. It is, therefore, most relevant for general collateral (GC) repos, which are typically thought

³Disputes and court investigations continue long time after the detection of the manipulation (Bloomberg, April 04, 2016, *Five Ex-Barclays traders plead not guilty to Libor manipulation*).

⁴See Financial Times, April 25, 2014, *US regulators urge quick Libor replacement*, retrieved from FT.com on April 29, 2014. In 2017, the Financial Conduct Authority announced that Libor will be phased out by the end of 2021 (Bloomberg, *Libor Funeral Set for 2021 as FCA Abandons Scandal-Tarred Rate*, July 27, 2017).

of as being driven by this objective. In GC repos, the cash taker (borrower) may deliver one of several securities to the cash provider (lender) from a prescribed basket, or list, of eligible collateral. For example, in the popular GC Pooling ECB basket on Eurex Repo (Figure 1a), the list of eligible collateral stood at approximately 7,500 ISINs in August 2013. It is the feature of a repo that the underlying collateral is made available to the cash provider that makes a repo different from a plain collateralized loan. This is also a key ingredient in our theory, but for a different reason than in models of special repo rates. In particular, in our model, the cash provider needs the underlying collateral to (partially) finance the reverse repo. To explain this, it is useful to first summarize our basic theoretical framework.

The perspective we take in this paper is that, apart from borrowing unsecured, the alternative to raising liquidity by doing repo is to sell the underlying security in the cash market and buy it back later. We can think of this as a “home-made” repo. This may involve additional transaction costs, for example due to illiquidity. In addition, in a home-made repo, the interest cost of raising the liquidity is a function of the security’s future price, which is stochastic. So the relative attractiveness of doing a regular or a home-made repo is a function of the repo rate in comparison to the illiquidity and risk-adjusted cost of engaging in cash market trades. Nyborg and Östberg (2014) provide empirical support for the idea that the security cash market is an important alternative source of liquidity to banks, especially when the unsecured market is tight.

An important ingredient in our model is that both the cash taker and the cash provider are constrained in the unsecured market. As our model abstracts from credit risk, the absence of borrowing constraints in the unsecured market would imply that the repo rate would always be equal to the unsecured rate. We abstract from credit risk for both theoretical and empirical reasons. First, from a theoretical perspective, credit risk cannot explain negative collateral spreads. Second, from an empirical perspective, as seen in Figure 1, negative collateral spreads are common in overnight CCP repos, where credit risk should not be a concern. Our model allows for the cash provider and taker to face different borrowing constraints. While it may seem “natural” to think of the cash taker to be more constrained, we study both the case that she is and the case that she is not,

which adds to the richness of the predictions of our theory.

One often thinks of cash providers in repos as liquidity rich. However, in our model, the cash provider, being constrained, may be thought of as a bank acting as an intermediary in the market for liquidity. In practice, unsecured borrowing constraints may arise for a number of reasons, for example, from search costs or regulation. At any point in time, there is a fixed amount of liquidity in the system, and there is evidence that there is a degree of allocational inefficiency in the market for liquidity even during times of normalcy (Bindseil, Nyborg, and Strebulaev, 2009).

Another feature of our model is that we allow for the possibility that the price of the collateral that is used in the repo is different from the price that would be realized in the security cash market. Such “pricing errors” may occur in practice, for example, because the security is not perfectly liquid. This affects the relative interest costs of a regular versus home-made repo.

Constraints in the unsecured market and the need to trade in the security cash market gives rise to a constrained-arbitrage relation between the repo rate, the unsecured rate, and the collateral’s cash market adjusted (for risk and illiquidity) rate of return. The nature of this relation depends on a number of parameters such as the potentially different borrowing constraints of the different players, the haircut in the repo, the players’ risk aversion coefficients, and the volatility and illiquidity of the underlying collateral. The sign of the collateral spread depends on the relation between the unsecured rate and the illiquidity and risk-adjusted cash market rate of return of the underlying security. Roughly speaking, the collateral spread is positive if and only if the cash market adjusted rate of return is below that of the unsecured rate.

Thus, collateral spreads can go from the normal positive situation to negative if either (1) the unsecured rate drops sufficiently, or (2) securities prices fall sufficiently, implying an increase in the illiquidity and risk-adjusted security cash market rate of return. Scenario (1) helps explain the spikes seen in Figure 1. As is well known, unsecured rates usually spike either up or down at the end of reserve maintenance periods and up at the end of calendar months (see, e.g., Hamilton, 1996 for US evidence and Perez-Quiros and Mendizabal, 2006; Nautz and Offermanns, 2008; Fecht, Nyborg, and Rocholl, 2008, for

evidence from the euro area). This causes a move in the same direction in the collateral spread, since the securities market is not similarly affected by these calendar effects. Scenario (2) helps explain the prolonged periods of negative collateral spreads shown in Figure 1 as periods where securities markets were depressed. Our theory also predicts that reduced borrowing capacities in the unsecured markets put downward pressure on collateral spreads. The conditions under which positive and negative collateral spreads obtain are elaborated further upon in the body of the paper.

One other result we want to point out here is that negative collateral spreads are only possible if the cash provider is less constrained in the unsecured market than the cash taker. Thus, the fact that collateral spreads are often negative may be viewed as empirical confirmation of the commonly held notion that cash providers are players who have a comparative advantage in the unsecured market. This result arises because of the basic mechanism of a repo with constrained players. If the collateral spread is negative, then it is advantageous to borrow as much as possible in the unsecured market. If the potential cash provider is less constrained than the player that seeks liquidity, then the latter can, essentially, relax her unsecured borrowing constraint by raising funds from the former. If the potential cash provider is more constrained, however, there are no such gains from trade. So in this case, a negative collateral spread would not be consistent with equilibrium.

We use data provided by Eurex Repo to test three predictions of our theory. For all three tests, we employ the two GC baskets with the most active overnight trading on the Eurex platform, namely the GC Pooling ECB basket and the GC Pooling Extended basket.⁵ The collateral spreads in Figure 1 are also based on these two baskets.

In the first test, we exploit the feature of the unsecured market that it spikes, up or down, at the end of the reserve maintenance period. This relates to the central bank's operational framework. Thus, by the arbitrage relation discussed above, the repo rate would be predicted to move less than the unsecured rate. As a result, the collateral spread is predicted to spike in the same direction as the unsecured rate. The evidence is strongly

⁵See Section 4 for details on these and other GC contracts trading on Eurex.

in support of our theory.

In the second test, we use an exogenous change to haircuts to test the prediction of our model that the collateral spread is decreasing in the haircut (when the collateral spread is positive). This uses the institutional feature of our data that in the two GC contracts we look at, Eurex, not the counterparties, determines the haircuts. This corresponds to an assumption in our model that haircuts are exogenous. Moreover, Eurex uses the same haircuts as in Eurosystem repos (Mancini, Ranaldo, and Wrampelmeyer, 2016; Nyborg, 2016), which, as shown by Nyborg (2016), have historically been updated every three to four years.⁶ On September 27, 2013, the ECB announced changes to haircuts in Eurosystem repos as of October 1, 2013. For the most part, haircuts were lowered. We have obtained actual haircuts from Eurex around this time and show that haircuts for the securities in the two baskets we are studying fell on October 2, 2013, consistent with Eurex' policy of using Eurosystem repo haircuts. Our tests show that collateral spreads also fell, as predicted by our theory.

In the third test, we examine the effect of volatility on collateral spreads. Our theory predicts that higher security cash market volatility is associated with a lower collateral spread. The intuition is that the higher volatility feeds into a higher repo rate because it becomes more "costly" for a risk averse cash provider to (partially) finance the reverse position through the cash market. Hence he requires a higher repo rate as compensation. To examine this, we study the change in the collateral spread on days with ECB Governing Council meetings where the policy rate is subject to change (typically every second Governing Council meeting). The securities in the two baskets we study are bonds of various kinds. As bond return volatility is closely linked to changes in interest rates, the uncertainty before a Governing Council meeting, in which they decide on a change in the policy rate, is higher than on the day itself, when the decision is announced. Thus, our model predicts that the change in the collateral spread is negative, which is also what we find in the data.

⁶The Eurosystem is comprised of the European Central Bank (ECB) and euro area member states' national central banks.

These findings lend support to our theory and thus to the explanation for negative collateral spreads that emerge from it. Thus, we end the paper by providing an analysis of the development of the collateral spread in the euro area, as shown in Figures 1a and 1b, over time. In this analysis, we also draw on developments in ECB unconventional monetary policies.

The rest of the paper is organized as follows. In Section 2, we lay out the theoretical framework. Section 3 contains the theoretical analysis of positive and negative collateral spreads and derives empirical predictions. The empirical tests are in Section 4. Section 5 discusses the collateral spread over time, and Section 6 concludes.

2 Theoretical framework

This section provides the theoretical framework that we will use to study variations in the collateral spread, the difference between unsecured and repo rates. We are especially interested in understanding under what conditions negative collateral spreads can arise and what are the other empirical implications of a model that is capable of yielding this. Important features in our setup include liquidity constrained players and collateral “pricing errors” (as discussed in the Introduction and further described below). To provide context for the model, we start by reviewing a generic repo.

2.1 Generic repurchase agreement

A generic repurchase agreement between two counterparties, a cash taker (borrower) and a cash provider (lender), has five main ingredients; the underlying collateral (e.g. a security), the price of the underlying collateral, the haircut that is applied to this price, the repo rate, and the maturity (tenor) of the repo agreement. For example, if the price (for the purpose of a repo) of the collateral is P , the haircut is h , and the repo rate is r , then the cash taker delivers the underlying collateral to the cash provider and receives cash of $P(1 - h)$. At maturity, the cash taker buys back the underlying collateral at a price equal to P plus the accumulated interest at the repo rate. The cash taker is said to be doing a

Table 1: *Basic cash flows in a repo/reverse repo**

	Date 0	Date 1 (maturity)
Repo (cash taker)	$1 - h$	$-(1 - h)(1 + r)$
Reverse (cash provider)	$-(1 - h)$	$(1 - h)(1 + r)$

* The price, P , of the collateral is normalized to 1 and the probability of default is assumed to be 0.

repo while the cash provider is said to be doing a reverse repo.

In addition, the cash provider typically obtains the use of the collateral until maturity. If we ignore this feature as well as the possibility of default, the cash flows arising from a repo that starts at date 0 and runs until date 1 are described in Table 1. These flows make the repo look like a simple (collateralized) loan of $1 - h$ at the repo rate, r (with the price normalized to 1). However, this ignores that the cash provider has use of the collateral until maturity. In our model, this feature will play an important role.

2.2 Further structure and assumptions

We consider a setup where one agent, that we think of as a bank and refer to as “the short,” is short one unit of liquidity. The shortage may arise from a need to fulfill reserve requirements, satisfy regulatory liquidity constraints, buy securities, or fulfill some obligation. The need to obtain the liquidity is modeled as a hard constraint. The short is assumed to be endowed with one unit of a security that can be used as collateral in a repo. She can obtain liquidity by selling the security in the cash market, doing a repo, or borrowing in the unsecured market. In the case the short does a repo, we refer to her as the cash taker, in line with standard terminology. Her potential counterparty is referred to as the cash provider.

We think of the cash provider as an intermediary between the short and banks with excess liquidity. It could be another bank. As a normalization, the cash provider is assumed to have no cash on hand at the start of date 0. Both the short and the cash provider are constrained in the unsecured market so that the total unsecured sum they can raise is less

than the unit the short needs. This means that it is not feasible for the cash provider to finance a reverse repo that will provide the short with the liquidity she needs in the unsecured market. A fraction of the collateral held by the short will have to be sold in the cash market, either by the short herself or, in the case she engages in a repo, by the cash provider. Constraints in the unsecured market are necessary to generate nonzero collateral spreads. Such constraints could arise for a number of reasons, for example, search costs. This assumption will also lead to a link between the unsecured rate, the repo rate, and the cash market rate of return of the underlying collateral. We refer to a repo rate as an *equilibrium* repo rate if the short and the cash provider are willing to undertake a repo at that rate.

Assumption 1. *The combined amount the short and the cash provider can obtain in the unsecured market is strictly less than one (i.e., the quantity of liquidity the short needs). That is, $0 < \eta + \kappa < 1$, where $\eta \in (0, 1)$ and $\kappa \in (0, 1)$ are the short and cash provider's borrowing capacities in the unsecured market, respectively.*

The assumption of constraints in the unsecured market is consistent with anecdotal evidence that banks face interbank credit limits. It is also consistent with the findings of Bindseil, Nyborg, and Strebulaev (2009) that the interbank market is not allocationally efficient, even during times of normalcy. The unsecured rate is denoted by u and is assumed to be the same for the short and the potential cash provider.

To put further structure on the analysis, we make several additional assumptions as listed below.

Assumption 2. *There is no default risk.*

This is assumed in part because we wish to keep the theoretical analysis as simple as possible. It is also difficult to see how default, or credit, risk can lead to negative collateral spreads. The assumption of no default risk means that the theoretical analysis in this paper should perhaps best be viewed as representing one of overnight or CCP transactions, where default risk is arguably minimal.

Assumption 3. *Haircuts are exogenous to the repo itself. That is, the haircut, $h \in [0, 1)$, is not subject to negotiation between the counterparties.*

This reflects the factual situation in many repo agreements that haircuts are set in advance and often by a third party. For example, in the case of Eurex’ repo contracts, a list of haircuts for each day is made available (weekly) on the Eurex website and (daily) in their system, Xemac. This is not updated during the day, except in special circumstances. As also noted by Mancini, Ranaldo, and Wrampelmeyer (2016) and Nyborg (2016), haircuts in Eurex’ GC Pooling contracts (that we use in our empirical analysis) are based on those set by the ECB for Eurosystem repos. These are updated only every three to four years (Nyborg, 2016).⁷ For US triparty repo agreements, Krishnamurthy, Nagel, and Orlov (2014) have observed that haircuts are not managed actively either and, according to Copeland, Martin, and Walker (2010), “...haircuts are not negotiated at the trade level but are instead written into the appendix of the tri-party repo custodial agreement between the cash investor, the collateral provider, and the clearing bank. While it is possible to change the appendix containing the haircuts, the change may not apply until the next day. Such changes are only made occasionally.”⁸ In Europe, Clearstream and Euroclear, two triparty repo agents, ask banks to set their own haircuts according to certain security criteria (e.g. type of security, maturity, rating etc.) before they start trading repo in their systems. This list of haircuts can be amended and banks can apply additional margins in individual contracts. However, haircuts are not normally updated on a daily basis. Thus, we study repo rates and collateral spreads as a function of haircuts.

Assumption 4. *The date 0 price, for the purpose of the repo, of the underlying security (collateral) is normalized to 1. The actual (security) cash market price that a seller could obtain is $1 - \varepsilon_0$, where $\varepsilon_0 < 1$ is a constant. At date 1, the cash market price of the underlying security is $1 + \tilde{x} - \varepsilon_1$, where ε_1 is a constant and \tilde{x} is a random variable*

The parameter ε_t is the collateral pricing error arising from a lack of perfect liquidity

⁷Nyborg (2016) also documents that Eurex deviates from the ECB haircuts in about 10% of cases.

⁸It is possible that haircuts in the bilateral repo market may be updated more frequently than for other repos, see e.g., (Gorton and Metrick, 2011).

or, at date 0, because the price used in the repo is based on a model. Even if a market price were used in a repo agreement, in practice, it is not clear that if additional securities were sold into the market, that the cash taker could actually achieve that price. The less liquid the underlying security is, the larger would the price discrepancy be expected to be. Because the ε 's reflect illiquidity, we think of $\varepsilon_t \geq 0$. We assume that settlement in the cash market is immediate.⁹

Unsecured borrowing capacities

We make two further assumptions regarding the players' unsecured borrowing constraints.

Assumption 5. *The unsecured borrowing capacities of both the short and the cash provider, η and κ , respectively, exceed the haircut, h , and the date 0 pricing error, ε_0 .*

This ensures that the cash provider is able to finance a reverse repo and that the short is able to raise the requisite unit of liquidity either through a regular or home-made repo.

A final issue with respect to the players' borrowing capacities is whether these are linked or not. Our baseline assumption is that they are linked:

Assumption 6. *If the short draws down on her unsecured borrowing capacity, the unsecured borrowing capacity of the cash provider is reduced by the same amount.*

This can be thought of as reflecting (unmodeled) linkages in the unsecured market for liquidity. It is analytically equivalent to assuming that, in a regular repo (combined with unsecured borrowing), the short obtains the full unit of liquidity she needs from the single, modeled cash provider. If $\eta > \kappa$, the possibility arises that the short borrows more than the borrowing capacity of the potential cash provider, turning his effective capacity negative. The interpretation would be that the banks outside the model call in loans that the cash provider has outstanding or reduce credit lines. An alternative to Assumption 6 would be that the two players' borrowing capacities are not linked. We discuss how this affects the analysis and the results in Subsection 3.6.

⁹If settlement is instead the next day, for example, we could think of the repo studied here as a tomorrow-next transaction.

Returns and preferences

Let $1 + \bar{x}$ denote the expected date 1 price of the underlying collateral in a perfectly liquid market without pricing errors. Since the date 0 price is normalized to 1, \bar{x} is the expected rate of return in these perfect conditions. In contrast, the actual expected rate of return to an agent that buys in the cash market at date 0 and sells at date 1 is

$$\bar{y} = \frac{1 + \bar{x} - \varepsilon_1}{1 - \varepsilon_0} - 1 = \frac{\bar{x} - (\varepsilon_1 - \varepsilon_0)}{1 - \varepsilon_0}. \quad (1)$$

Similarly, we define the random variable

$$\tilde{y} \equiv \frac{\tilde{x} - (\varepsilon_1 - \varepsilon_0)}{1 - \varepsilon_0}. \quad (2)$$

This represents the (security) cash market rate of return.

The short's objective is to raise one unit of liquidity at date 0 in the way that yields the maximum date 1 utility. The cash provider in a repo seeks to provide liquidity while also maximizing date 1 utility.

Assumption 7. *The short and the cash provider have CARA utility with risk aversion parameter $\rho \geq 0$, and \tilde{x} is normally distributed with mean \bar{x} and variance σ_x^2 .*

The cash market rate of return, \tilde{y} , is, therefore, normally distributed with mean \bar{y} and variance $\sigma_y^2 = \sigma_x^2 / (1 - \varepsilon_0)^2$. As shown by Grossman (1976), Assumption 7 leads to mean-variance preferences. For the analysis in the next section, it is useful to make some observations regarding the certainty equivalents of the returns obtained from various positions or trades of the security held by the short.

Position certainty equivalents

Using Grossman's (1976) arguments, we can establish that the certainty equivalent of receiving $\omega\tilde{x}$, $\omega \in [0, 1]$, is $\omega\bar{x} - \frac{\rho}{2}\sigma_x^2\omega^2$, while the certainty equivalent of an outflow of $\omega\tilde{x}$ is¹⁰

¹⁰Let $\tilde{z} = \omega\tilde{x}$. Thus, \tilde{z} is normally distributed with mean $\bar{z} = \omega\bar{x}$ and variance $\sigma_z^2 = \omega^2\sigma_x^2$. The expected utility of receiving $\omega\tilde{x}$ is given by,

$$E[U(\omega\tilde{x})] = E[U(\tilde{z})] = \frac{-1}{\sqrt{2\pi}\sigma_z} \int_{-\infty}^{\infty} \exp(-\rho z) \exp\left(\frac{-(z - \bar{z})^2}{2\sigma_z^2}\right) dz = -\exp\left(-\rho\left(\bar{z} - \frac{\rho}{2}\sigma_z^2\right)\right),$$

$$\hat{x}(\omega) \equiv \bar{x}\omega + \frac{\rho}{2}\sigma_x^2\omega^2. \quad (3)$$

That is to say, an agent who has to pay $\omega\tilde{x}$ would be indifferent between paying this random sum or the fixed sum $\hat{x}(\omega)$. In general, given Assumption 7, the certainty equivalent of an outflow of $\omega\tilde{x} + a$, where a is a constant, is $\hat{x}(\omega) + a$.

Cash market certainty equivalents

The alternative to raising one unit of liquidity by doing a repo and borrowing h in the unsecured market is to sell ω units of the security in the security cash market, borrow $1 - \omega(1 - \varepsilon_0)$ in the unsecured market, and then buying back ω units of the security at date 1. An agent that follows this strategy, has a net *outflow* of $\omega[\tilde{x} - (\varepsilon_1 - \varepsilon_0)]$ from the two cash market trades. The certainty equivalent of this is $\hat{x}(\omega) - \omega(\varepsilon_1 - \varepsilon_0)$. In other words, *per unit of cash raised at date 0*, the certainty equivalent of the net cash *outflow* from selling fraction ω of the security at date 0 and buying this back at date 1 is¹¹

$$\hat{y}(\omega) \equiv \frac{\hat{x}(\omega) - \omega(\varepsilon_1 - \varepsilon_0)}{\omega(1 - \varepsilon_0)} = \frac{\bar{x} - (\varepsilon_1 - \varepsilon_0)}{1 - \varepsilon_0} + \frac{\rho}{2} \frac{\omega\sigma_x^2}{1 - \varepsilon_0} = \bar{y} + \frac{\rho}{2}\omega\sigma_y^2(1 - \varepsilon_0). \quad (4)$$

We refer to $\hat{y}(\omega)$ as the “adjusted rate of return,” or “cost,” from selling the fraction ω of the security at date 0 and buying it back at date 1. This terminology reflects that $\hat{y}(\omega)$ is adjusted relative to the fundamental expected rate of return of the security, \bar{x} . As seen, $\hat{y}(\omega)$, is increasing in the fraction traded, ω . The additional adjustments are, in part, due to risk aversion and volatility and, in part, to illiquidity. In particular, the cash market adjusted rate of return from selling ω shares at date 0 and buying this back at date 1 is also increasing in risk aversion (ρ), volatility (σ_x^2), and illiquidity (ε_0). As seen in (4), $\hat{y}(\omega)$ is also increasing in \bar{x} . Finally, note that we assume that u and \bar{y} strictly exceed -1 .

where $E[\cdot]$ is the expectation operator, $U(\cdot)$ denotes the negative exponential (CARA) utility function, i.e. $U(z) = -\exp(-\rho z)$, and $\exp(z) \equiv e^z$. Thus the certainty equivalent of receiving $\omega\tilde{x}$ is $\omega\bar{x} - \frac{\rho}{2}\omega^2\sigma_x^2$. Similarly, $E[U(-\omega\tilde{x})] = -\exp(-\rho(\omega\bar{x} + \frac{\rho}{2}\omega^2\sigma_x^2))$, implying that the certainty equivalent of an outflow of $\omega\tilde{x}$ is $\hat{x}(\omega)$ as defined in (3).

¹¹The second step in (4) uses (3) and the final step uses (1) and the expression for σ_y^2 above.

3 Analysis

In our model, constraints in the unsecured market mean that at least a fraction of the security held by the short will have to be sold in the cash market. This may be done by the short herself or, in the case that she raises liquidity through a repo, by the cash provider in that repo. Because the cash provider is also constrained in the unsecured market and does not have excess liquidity, he needs to finance the reverse position by selling securities at date 0. Thus, we study reverse repos where the cash provider needs to sell at least a fraction of the underlying repoed collateral to finance his position. The existence of constraints in the unsecured market for both players and the need to trade in the cash market creates a link between the repo rate, the unsecured rate, and the collateral's cash market adjusted rate of return. In this section, we work out the nature of this constrained-arbitrage link and its implications.

We consider a regular repo and a “home-made repo” as discrete alternatives to raise liquidity. Thus, the basic tradeoff between a repo and cash market trades is studied on a discrete margin. The same basic tradeoff can be studied on an infinitesimal margin by allowing continuous mixtures of repos, cash market trades, and unsecured borrowing. Since the tradeoff is fundamentally the same, the results would be similar to what we derive here. We think the discrete approach has some advantage with respect to realism and clarity of exposition. However, for both the regular and home-made repo alternatives, we derive the optimal mixture between repo or cash-market trading, on the one hand, and unsecured borrowing, on the other.

The short, then, has two alternative sets of trades. She can either combine unsecured borrowing with a repo (Alternative 1) or with direct cash market trades (Alternative 2). In Alternative 1, the short clearly prefers minimizing her unsecured borrowings if $r \leq u$ and maximizing them if $r > u$.¹² In Alternative 2 (home-made repo), the optimal cash market trade and unsecured borrowings will have to be determined.

¹²If $r = u$, the short is indifferent between raising liquidity through a repo or unsecured borrowing. In the analysis, we assume that the short borrows as little as possible in the unsecured market if $r = u$. Thus, we have two scenarios: $r \leq u$ and $r > u$.

Table 2: *Cash flows from alternatives for raising one unit of liquidity*

	Date 0	Date 1
<i>Alternative 1 (when $r \leq u$)</i>		
Repo	$1 - h$	$-(1 - h)(1 + r)$
Borrow unsecured	h	$-h(1 + u)$
Sum	1	$-[1 + (1 - h)r + hu]$
<i>Alternative 2 ("home-made repo")</i>		
Sell	$\omega(1 - \varepsilon_0)$	-
Buy	-	$-\omega(1 + \tilde{x} - \varepsilon_1)$
Borrow unsecured	$1 - \omega(1 - \varepsilon_0)$	$-(1 - \omega(1 - \varepsilon_0))(1 + u)$
Sum	1	$-\omega(1 + \tilde{x} - \varepsilon_1) - (1 - \omega(1 - \varepsilon_0))(1 + u)$

By way of illustration, the cash flows from the short's two alternatives are laid out in Table 2, assuming a positive collateral spread. Given $r \leq u$, under Alternative 1, the short optimally chooses to borrow the minimum amount, h , in the unsecured market and raise $1 - h$ by repoing her security. In Alternative 2, the short sells ω units in the cash market, yielding a cash inflow of $\omega(1 - \varepsilon_0)$, and borrows $1 - \omega(1 - \varepsilon_0)$ at the unsecured rate. At date 1, she buys back ω shares of the security and repays her loan, to yield the cash flows shown. In order to compare the two alternatives, it is necessary to derive the optimal ω , subject to constraints, which we turn to next.

3.1 Alternative 2: Raising liquidity in the cash market

As seen in Table 2, the outflow at date 1 from the short's Alternative 2 is

$$\omega(1 + \tilde{x} - \varepsilon_1) + (1 - \omega(1 - \varepsilon_0))(1 + u).$$

Using (3), the certainty equivalent of this is

$$1 + \hat{x}(\omega) - \omega(\varepsilon_1 - \varepsilon_0) + (1 - \omega(1 - \varepsilon_0))u. \quad (5)$$

Using (4), this can be written as $1 + c(\omega)$, where

$$c(\omega) \equiv \omega(1 - \varepsilon_0)\hat{y}(\omega) + (1 - \omega(1 - \varepsilon_0))u. \quad (6)$$

$c(\omega)$ has the intuitive interpretation as the (adjusted) weighted average cost of liquidity under Alternative 2 when ω shares are sold in the cash market at date 0 and the remaining liquidity of $1 - \omega(1 - \varepsilon_0)$ is obtained in the unsecured market.

Thus, under Alternative 2, maximizing date 1 utility for the short is equivalent to choosing ω so as to minimize the weighted average cost of liquidity. In doing so, the short faces two constraints. First, it is not feasible for her to sell more of the security than the one unit she is endowed with. Second, since she cannot borrow more than η unsecured, she must sell at least $(1 - \eta)/(1 - \varepsilon_0)$ units. Hence, the short's problem is to solve the following constrained minimization problem:

$$\begin{aligned} \min_{\omega} c(\omega) \quad & \text{subject to} \\ \text{Feasibility:} \quad & \omega \leq 1 \\ \text{Unsecured borrowing:} \quad & \omega \geq \frac{1-\eta}{1-\varepsilon_0}. \end{aligned} \quad (7)$$

The first-order condition of the *unconstrained* problem is¹³

$$\omega^* \hat{y}'(\omega^*) + \hat{y}(\omega^*) - u = 0. \quad (8)$$

Thus, using (4), the unconstrained optimal cash market trade is

$$\omega^* = \frac{u - \bar{y}}{\rho\sigma_y^2(1 - \varepsilon_0)}. \quad (9)$$

Hence, with respect to feasibility, $\omega^* \leq 1$ if and only if

$$u - \bar{y} \leq \rho\sigma_y^2(1 - \varepsilon_0). \quad (10)$$

With respect to the unsecured borrowing constraint, $\omega^* \geq (1 - \eta)/(1 - \varepsilon_0)$ if and only if

$$u - \bar{y} \geq \rho\sigma_y^2(1 - \eta). \quad (11)$$

¹³From (4), it is straightforward that $c''(\omega^*) > 0$ (assuming $\rho > 0$ and $\sigma_y^2 > 0$). So the second order condition for a minimum is satisfied. If either ρ or σ_y^2 equal zero, it is straightforward that $\omega^* = 1$ if $\bar{y} \leq u$ and $\frac{1-\eta}{1-\varepsilon_0}$ otherwise. In other words, either the feasibility or the borrowing constraint binds. The analysis below goes through, but without the intermediate case where neither of these constraints bind.

The “upper bound” on $u - \bar{y}$ (for an unconstrained solution) in (10) is larger than the “lower bound” in (11) since, by assumption, $\eta \geq \varepsilon_0$ (feasibility of Alternative 2). Thus, the constrained optimal cash market trade is

$$\Omega = \begin{cases} 1 & \text{if } u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0) \\ \omega^* & \text{if } \rho\sigma_y^2(1 - \eta) \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0) \\ \frac{1-\eta}{1-\varepsilon_0} & \text{if } u - \bar{y} < \rho\sigma_y^2(1 - \eta). \end{cases} \quad (12)$$

This says that if the unsecured rate is “very large” relative to the cost of cash market trades, the short optimally trades her whole unit. On the other hand, if the unsecured rate is “very low,” the short trades as little as possible, preferring instead to borrow as much as she can in the unsecured market. Between these extremes, the unconstrained optimum obtains. In the limiting case that $\rho = 0$, $\Omega = 1$ if and only if $\bar{y} \leq u$, which is intuitive.

3.2 Positive collateral spread

In this subsection, we consider the case of positive collateral spreads. Negative spreads are considered in the next subsection.

We start by assuming that $r \leq u$. Our first objective is to derive necessary conditions for this to hold. Sufficient conditions will also be covered. Given $r \leq u$, the short’s two alternative trades are as laid out in Table 2, with $\omega = \Omega$ as given by (12). Using the analysis in Subsection 3.1, it follows from Table 2 that for the short to engage in a repo, we must have,

$$(1 - h)r + hu \leq \Omega(1 - \varepsilon_0)\hat{y}(\Omega) + (1 - \Omega(1 - \varepsilon_0))u. \quad (13)$$

This intuitive condition says that the interest cost of doing a repo (in combination with unsecured borrowing) must be no larger than the cost of trading in the cash market (in combination with unsecured borrowing). In turn, this implies that the maximum repo rate the short is willing to accept is

$$r \leq \bar{r} \equiv \frac{\Omega(1 - \varepsilon_0)\hat{y}(\Omega) + (1 - \Omega(1 - \varepsilon_0))u - hu}{1 - h}. \quad (14)$$

Table 3: *Cash flows to the cash provider (when $r \leq u$)*

	Date 0	Date 1
Reverse	$-(1 - h)$	$(1 - h)(1 + r)$
Sell security	$\alpha(1 - \varepsilon_0)$	–
Unsecured loan	$1 - h - \alpha(1 - \varepsilon_0)$	$-(1 - h - \alpha(1 - \varepsilon_0))(1 + u)$
Buy back security	–	$-\alpha(1 + \tilde{x} - \varepsilon_1)$
Sum	0	$(1 - h)(1 + r) - (1 - h - \alpha(1 - \varepsilon_0))(1 + u) - \alpha(1 + \tilde{x} - \varepsilon_1)$

This expression is derived under the assumption that $r \leq u$. Thus, an upper bound on the repo rate is $\min\{\bar{r}, u\}$.

Turning now to the cash provider, recall that he needs to finance the reverse repo by selling collateral at date 0. This may be combined with a loan in the unsecured market. In order to be able to return the collateral to the short at date 1, the cash provider has to buy it back in the market at that time. At date 0, it is possible that the cash provider generates excess liquidity through the sale of the security in the cash market. If so, this is assumed to be placed in the unsecured market at u . The cash provider, therefore, faces cash flows shown in Table 3, where α denotes the fraction of the underlying security he sells in the cash market to generate liquidity and finance the reverse repo.

The cash flows in Table 3 imply that for the (potential) cash provider to be willing to enter into the reverse repo, we must have (using (3) and (4)),

$$(1 - h)r + hu \geq \alpha(1 - \varepsilon_0)\hat{y}(\alpha) + (1 - \alpha(1 - \varepsilon_0))u. \quad (15)$$

This is the reverse of the condition for which the short is willing to enter a repurchase agreement, but with α substituting for Ω . To derive the optimal fraction to sell in the cash market for the cash provider, note that the problem he faces is identical to the problem faced by the cash taker under her Alternative 2, except that the cash provider's unsecured borrowing cap is κ rather than η . Thus, using the same argument as in Subsection 3.1,

the cash provider's constrained optimal cash market trade is¹⁴

$$A = \begin{cases} 1 & \text{if } u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0) \\ \omega^* & \text{if } \rho\sigma_y^2(1 - \kappa) \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0) \\ \frac{1-\kappa}{1-\varepsilon_0} & \text{if } u - \bar{y} < \rho\sigma_y^2(1 - \kappa). \end{cases} \quad (16)$$

Equation (15) now implies that the minimum repo rate the cash provider is willing to accept is given by

$$r \geq \underline{r} \equiv \frac{A(1 - \varepsilon_0)\hat{y}(A) + (1 - A(1 - \varepsilon_0))u - hu}{1 - h}. \quad (17)$$

Comparing (17) to (14) shows that the minimum repo rate the cash provider is willing to accept, \underline{r} , coincides with the maximum the short is willing to pay, \bar{r} , if their respective unsecured borrowing capacities, κ and η , are identical. For in this case, their respective security cash market trades, A and Ω would be identical for all values of $u - \bar{y}$. More generally, if $\eta \neq \kappa$, \underline{r} and \bar{r} diverge if at least one of the players' borrowing constraints bind.

The expressions above have been derived under the assumption of a positive collateral spread, that is, $r \leq u$. So necessary conditions for a positive collateral spread are (i) $\underline{r} \leq u$ and (ii) $\underline{r} \leq \bar{r}$. By construction, it is clear that these two conditions are also sufficient for the existence of equilibrium $r \leq u$. We first address these conditions in the simple case that neither the short's nor the cash provider's unsecured borrowing constraints are binding.

Lemma 1. *If $u - \bar{y} \geq \rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\}$ then $\Omega = A$, $\bar{r} = \underline{r} \leq u$, and there is a unique equilibrium repo rate $r \leq u$ (as a function of parameter values). Specifically:*

¹⁴ Assumption 6 implies that the cash provider's unsecured borrowing capacity is reduced to $\kappa - h$ in the case of a repo, since the short draws h of unsecured borrowing. Thus, the unsecured borrowing constraint in the cash provider's problem becomes $\alpha(1 - \varepsilon_0) \geq 1 - h - (\kappa - h) = 1 - \kappa$. The term $1 - h$ reflects that the cash provider only has to provide $1 - h$ units of liquidity through the reverse repo. Using (9), we then obtain the associated expression for A in (16). With non-linked borrowing capacities, the unsecured borrowing constraint of the cash provider would be given by $\alpha(1 - \varepsilon_0) \geq 1 - h - \kappa$. See Subsection 3.6 for a discussion of the implications.

1. If $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)$ then

$$r = \frac{(1 - \varepsilon_0)\hat{y}(1) + \varepsilon_0u - hu}{1 - h} \leq u. \quad (18)$$

2. If $\rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\} \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0)$ then

$$r = \frac{\omega^*(1 - \varepsilon_0)\hat{y}(\omega^*) + (1 - \omega^*(1 - \varepsilon_0))u - hu}{1 - h} = u - \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1 - h)} \leq u. \quad (19)$$

Proof: See the Appendix.

The lemma shows that when borrowing constraints are not binding, the maximum repo rate the short is willing to pay, \bar{r} , and the minimum rate the cash provider is willing to accept, \underline{r} , coincide and are bounded above by u . Hence, this common rate is an equilibrium repo rate $r \leq u$. This also means that $u - \bar{y} \geq \rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\}$ is a sufficient condition for a positive equilibrium collateral spread. While the lemma is incomplete in that it does not consider binding borrowing constraints or the possibility of negative collateral spreads, it is suggestive of a positive collateral spread being associated with a “large” u relative to \bar{y} . As seen, what “large” means here depends on volatility, risk aversion, and the players’ unsecured borrowing capacities.

A general point of our analysis is that there is a constrained arbitrage relation between a (potential) equilibrium repo rate, r , the unsecured rate, u , and the cash market adjusted rate of return of the underlying security, \hat{y} . If \underline{r} and \bar{r} (and A and Ω) coincide, as they do in Lemma 1, this can be expressed as

$$(1 - h)r + hu = \Omega(1 - \varepsilon_0)\hat{y}(\Omega) + (1 - \Omega(1 - \varepsilon_0))u. \quad (20)$$

In words: the weighted average cost of liquidity raised via a regular repo equals the weighted average cost of liquidity raised via security cash market trading (home-made repo). Thus, when there is a unique repo rate, we also see that a positive collateral spread ($r \leq u$) requires that the risk and illiquidity adjusted expected rate of return of the underlying security (cost of liquidity in the security cash market) is less than the unsecured rate ($\hat{y}(\Omega) \leq u$). So, from the perspective of our analysis, one may interpret a positive collateral spread as indicative of unsecured borrowing being relatively expensive, or, equivalently, security prices being relatively high.

In general, movements in the unsecured rate or the price of the underlying security imply that the repo rate must also change so that the balance in (20) is maintained. This basic insight and its generalization (below), gives rise to a rich set of empirical predictions, as discussed further in Subsection 3.5.

When the unsecured borrowing constraint is binding for either player, the repo rate is indeterminate within the upper and lower bounds derived above, if $\underline{r} < \bar{r}$, or there is no equilibrium repo rate less than or equal to u . The exact conditions for a positive collateral spread depend on the players' unsecured borrowing capacities, as laid out in the following theorem.

Theorem 1.

1. *Suppose the short has a larger unsecured borrowing capacity than the cash provider, that is, $\eta > \kappa$. There is equilibrium $r \leq u$ if and only if borrowing constraints are not binding, that is,*

$$u - \bar{y} \geq \rho\sigma_y^2(1 - \kappa). \quad (21)$$

If this condition is met, there is a unique equilibrium repo rate $r \leq u$ (as a function of parameter values), as given in Lemma 1.

2. *Suppose the short has a smaller unsecured borrowing capacity than the cash provider, that is, $\eta \leq \kappa$. There is equilibrium $r \leq u$ if and only if the risk and illiquidity adjusted expected rate of return of the underlying security at the smallest possible volume to finance a reverse repo is at most equal to the unsecured rate, that is, if and only if*

$$\hat{y} \left(\frac{1 - \kappa}{1 - \varepsilon_0} \right) = \bar{y} + \frac{\rho}{2}\sigma_y^2(1 - \kappa) \leq u. \quad (22)$$

In this case:

- (a) *If $u - \bar{y} \geq \rho\sigma_y^2(1 - \eta)$, there is a unique equilibrium repo rate (as a function of parameter values) $r \leq u$, as given in Lemma 1.*

(b) If $u - \bar{y} < \rho\sigma_y^2(1 - \eta)$ then equilibrium $r \leq u$ is any $r \in [\underline{r}, R]$, where¹⁵

$$R = \min\{u, \bar{r}\} = \begin{cases} \bar{r} & \text{if } \rho\sigma_y^2(1 - \eta) > u - \bar{y} > \rho\sigma_y^2(1 - \eta)/2 \\ u & \text{if } u - \bar{y} \leq \rho\sigma_y^2(1 - \eta)/2, \end{cases} \quad (23)$$

$$\bar{r} = \frac{(1 - \eta)\hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right) + (\eta - h)u}{1 - h} = \frac{(1 - \eta) \left(\bar{y} + \frac{1}{2}\sigma_y^2(1 - \eta) \right) + (\eta - h)u}{1 - h}, \quad (24)$$

and

$$\underline{r} = \begin{cases} u - \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1 - h)} & \text{if } u - \bar{y} \geq \rho\sigma_y^2(1 - \kappa) \\ \frac{(1 - \kappa)\hat{y} \left(\frac{1 - \kappa}{1 - \varepsilon_0} \right) + (\kappa - h)u}{1 - h} = \frac{(1 - \kappa) \left(\bar{y} + \frac{1}{2}\rho\sigma_y^2(1 - \kappa) \right) + (\kappa - h)u}{1 - h} & \text{if } u - \bar{y} < \rho\sigma_y^2(1 - \kappa). \end{cases} \quad (25)$$

Proof: See the Appendix.

The theorem establishes that there is an equilibrium repo rate below the unsecured rate if and only if $u - \bar{y}$ is positive and bounded away from zero. The exact bound depends on which player is more constrained in the unsecured market. There is a discrete drop as we go from the short being less constrained ($\eta > \kappa$) to more constrained ($\eta \leq \kappa$). In other words, the range of $u - \bar{y}$ for which there is a positive collateral spread expands if the short's borrowing capacity falls below that of the cash provider.

The discontinuity in the lower bound on $u - \bar{y}$ for a positive collateral spread and, more generally, the link between the repo rate, the unsecured rate, and the security's expected rate of return arise partly because, for the short, the alternative to a regular repo is a home-made repo. The latter combines trade in the security cash market with unsecured borrowing. In addition, the cash provider in a regular repo needs to transact in the cash market in order to help finance the reverse position. Thus, the terms at which the players are willing to do a repo depends on conditions in the security cash market and the unsecured market. Unsecured borrowing capacities matter because they affect the players' constrained-optimal mixtures of unsecured borrowing and security trading.

¹⁵If $\eta = \kappa$ then 2(b) of Theorem 1 can be stated as: $\bar{r} = \underline{r}$ and there is a unique equilibrium repo rate (as a function of parameter values) given by the lower expression in (25).

If neither players' unsecured borrowing constraint is binding in their respective security cash market problems, there is a unique equilibrium repo rate which, as is intuitive, is independent of the players' unsecured borrowing capacities (Lemma 1).

However, if unsecured borrowing is so cheap that either of the borrowing constraints bind, the maximum repo rate the short is willing to pay, \bar{r} , and the minimum rate the cash provider requires, \underline{r} diverge. For example, if the short has a larger unsecured borrowing capacity, she is able to get closer to the unconstrained optimum (in the cash market problem) than the constrained cash provider. As a result, the maximum rate the short is willing to pay is lower than the minimum rate required by the cash provider. In other words, there is no equilibrium repo rate. Thus, when the short has the larger unsecured borrowing capacity, there is a positive equilibrium collateral spread if and only if the cash provider's borrowing constraint is not binding, that is, (21) in Theorem 1 holds. In this case, there is a unique equilibrium repo rate.

On the other hand, if the short has a smaller unsecured borrowing capacity, her borrowing constraint binds first and, when it does, we have $\bar{r} > \underline{r}$. The short is essentially willing to pay a premium over what the cash provider requires because her demand for unsecured funds is unsatisfied (the constraint is binding) in the cash market alternative. The cash provider can now funnel additional unsecured borrowings to the short through the device of the repo. In a way, the repo allows the players to arbitrage their borrowing constraints. This is mutually beneficial at $r \leq u$ as long as $\underline{r} \leq u$. The condition for this is just (22) in Theorem 1 and any rate in the interval $[\underline{r}, u]$ is equilibrium.

The condition (22) also represents the point where the risk and illiquidity adjusted cost of liquidity in the security cash market equals the unsecured rate when the cash provider borrows up to his capacity in the unsecured market. So this is an intuitive condition for a positive equilibrium collateral spread.

A way to think about what happens in the model is that a repo potentially allows a constrained short to expand her unsecured borrowing capacity through repoing with another player. This can reduce her total cost of liquidity if unsecured borrowing is cheap and if the counterparty can take advantage of this to a larger extent than the short. If the counterparty faces a tighter constraint in the unsecured market, the repo has no added

value, and a repo is only acceptable to both parties if the borrowing constraint of the counterparty does not bind. On the other hand, if the counterparty is less constrained in the unsecured market, then the repo can add value by allowing the short to implicitly expand her borrowing capacity. This also provides a simple intuition as to why the range over which there is a positive equilibrium collateral spread expands when the short is more constrained.

Under risk neutrality, the conditions for a positive equilibrium collateral spread collapse to $\bar{y} \leq u$. This can be understood by noting that if $\bar{y} > u$, the cash provider's cost of funding the reverse repo would exceed the unsecured rate. To break even, he therefore needs a repo rate above the unsecured rate. The short would be willing to pay such a rate because her home-made repo would also cost her more than u .

Our analysis provides constrained arbitrage relations between unsecured, security cash market, and repo rates. It is predicated on the idea that two of these rates may diverge, for reasons outside of the model. As they diverge, this then has implications for the third rate. We think of the unsecured market and the cash market as the fundamental markets, with the repo market being derived from those. Our analysis reflects that the two fundamental markets have different participants and are subject to different shocks. The participants in the unsecured markets are banks trading reserves (central bank money). The participants in the security cash market is much broader. As is well known, the unsecured market is subject to up and down spikes that relate to the shift from an outgoing to a new reserve maintenance period (see, e.g., Hamilton (1996), for the US, or Nautz and Offermanns (2008), for the euro area) or particular calendar dates, e.g., (Fecht, Nyborg, and Rocholl, 2008; Perez-Quiros and Mendizabal, 2006). Securities markets do not experience the same extreme movements around the same dates and are subject to a different set of issues such as investors' rebalancing portfolios, information flows, market uncertainty, etc. Given that the unsecured rate does not move in lock-step with the security cash market, the expressions for the repo rate in Theorem 1 will give rise to testable empirical predictions. However, we first need to study the case of negative collateral spreads.

3.3 Negative collateral spread

We now turn to necessary and sufficient conditions for a negative equilibrium collateral spread. We start by assuming $r > u$. In this case, the short optimally borrows her maximum of η in the unsecured market under Alternative 1 (repo) and repos the fraction

$$\phi = \frac{1 - \eta}{1 - h} \quad (26)$$

of her security. Thus, under the repo alternative, the short's cost of liquidity is

$$(1 - \eta)r + \eta u. \quad (27)$$

Furthermore, since the short faces the same situation in the cash market as before, her optimal transaction under Alternative 2 is still Ω as given in (12). These observations allow us to establish:

Lemma 2. *There is equilibrium $r > u$ only if*

$$\hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right) = \bar{y} + \frac{\rho}{2} \sigma_y^2 (1 - \eta) > u. \quad (28)$$

In particular, this implies that (i) the short's unsecured borrowing constraint (under Alternative 2) is binding, and (ii) if $\eta > \kappa$, the cash provider's borrowing constraint is also binding, that is,

$$u - \bar{y} < \rho \sigma_y^2 (1 - \kappa). \quad (29)$$

Proof: See the Appendix.

The lemma says that a negative collateral spread does not only mean that the unsecured rate is low relative to the repo rate, but also that it is low relative to the expected rate of return of the underlying security. In fact, the unsecured rate is so low that borrowing constraints are binding. Put differently, unsecured borrowing is cheap relative to all alternatives.

Since the short's borrowing constraint must be binding, under Alternative 2 she transacts $\Omega = (1 - \eta)/(1 - \varepsilon_0)$ in the security cash market and borrows η at the unsecured rate.

Therefore, along the same lines as in the derivation of (13), for the short to be willing to do a repo at $r > u$, we must have

$$(1 - \eta)r + \eta u \leq (1 - \eta)\hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right) + \eta u. \quad (30)$$

Hence, the maximum repo rate the short is willing to pay is

$$r \leq \bar{r}_{neg} \equiv \hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right) = \bar{y} + \frac{\rho}{2} \sigma_y^2 (1 - \eta). \quad (31)$$

The cash provider's problem is essentially the same as when $r \leq u$, except that he now needs to provide $1 - \eta$ of liquidity against $\phi < 1$ units of the underlying security. The cash flows from his trades are identical to those in Table 3, with η in place of h . The borrowing constraint remains the same as before, by Assumption 6. However, the feasibility constraint changes because the cash provider cannot sell more than the ϕ units he gets in the repo. Thus, the cash provider's optimal trade is now

$$A = \begin{cases} \phi & \text{if } u - \bar{y} \geq \rho \sigma_y^2 (1 - \varepsilon_0) \phi \\ \omega^* & \text{if } \rho \sigma_y^2 (1 - \kappa) \leq u - \bar{y} \leq \rho \sigma_y^2 (1 - \varepsilon_0) \phi \\ \frac{1 - \kappa}{1 - \varepsilon_0} & \text{if } u - \bar{y} \leq \rho \sigma_y^2 (1 - \kappa). \end{cases} \quad (32)$$

As written, (32) assumes that the feasibility constraint does not trivially bind, that is, $(1 - \varepsilon_0)\phi \geq (1 - \kappa)$. In fact, this is more than an assumption. It is a necessary condition for equilibrium $r > u$.

Lemma 3. *Equilibrium $r > u$ implies that the cash provider's feasibility constraint does not trivially bind, that is, $(1 - \varepsilon_0)\phi \geq (1 - \kappa)$.*

Proof: See the Appendix.

The intuition for this is straightforward: if $r > u$, it is desirable to borrow as much as possible unsecured. The flip side is that it is desirable to sell as little of the underlying security as possible. For a different intuition: The feasibility constraint gives the maximum units the cash provider can sell. The borrowing constraint gives the smallest quantity he needs to sell in order to finance the reverse repo. Thus, equilibrium requires the borrowing constraint to be below the feasibility constraint.

Since the cash provider extends η in liquidity to the short in the repo (rather than h as when $r \leq u$), the condition for him to be willing to do a repo becomes

$$(1 - \eta)r + \eta u \geq A(1 - \varepsilon_0)\hat{y}(A) + (1 - A(1 - \varepsilon_0))u. \quad (33)$$

Thus, the lowest acceptable repo rate to the cash provider is

$$r \geq \underline{r}_{neg} \equiv \frac{A(1 - \varepsilon_0)\hat{y}(A) + (1 - A(1 - \varepsilon_0))u - \eta u}{1 - \eta}. \quad (34)$$

To check whether $r > u$ is consistent with equilibrium, we need to compare this lower bound on r with the maximum the short is willing to pay, as derived above. We also need to check that $\bar{r}_{neg} > u$.

Theorem 2.

1. *Suppose the short has a smaller unsecured borrowing capacity than the cash provider, that is, $\eta \leq \kappa$. Suppose also that $(1 - \varepsilon_0)\phi \geq 1 - \kappa$. There is equilibrium $r > u$ if and only if (28) holds, that is, $u - \bar{y} < \frac{1}{2}\rho\sigma_y^2(1 - \eta)$. When this holds, $\bar{r}_{neg} > u$. Furthermore, if $\eta < \kappa$, $\underline{r}_{neg} < \bar{r}_{neg}$ and equilibrium $r > u$ may take on any value in the interval $(R_{neg}, \bar{r}_{neg}]$, where $R_{neg} = \max\{u, \underline{r}_{neg}\}$, and*

$$\underline{r}_{neg} = \begin{cases} \frac{\phi(1 - \varepsilon_0)\hat{y}(\phi) + (1 - \phi(1 - \varepsilon_0))u - \eta u}{1 - \eta} & \text{if } u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)\phi \\ u - \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1 - \eta)} & \text{if } \rho\sigma_y^2(1 - \kappa) \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0)\phi \\ \frac{(1 - \kappa)\hat{y}\left(\frac{1 - \kappa}{1 - \varepsilon_0}\right) + (\kappa - \eta)u}{1 - \eta} & \text{if } u - \bar{y} < \rho\sigma_y^2(1 - \kappa). \end{cases} \quad (35)$$

If (28) holds and $\eta = \kappa$ then $\bar{r}_{neg} = \underline{r}_{neg}$ and there is a unique equilibrium $r > u$, namely, $r = \hat{y}\left(\frac{1 - \eta}{1 - \varepsilon_0}\right) = \bar{y} + \frac{1}{2}\rho\sigma_y^2(1 - \eta)$.

2. *Suppose the short has a larger unsecured borrowing capacity than the cash provider, that is, $\eta > \kappa$. There does not exist equilibrium $r > u$.*

Proof: See the Appendix.

The theorem establishes that there is a negative equilibrium collateral spread only if the short has a smaller unsecured borrowing capacity than the cash provider, ($\eta \leq \kappa$) and

the latter's feasibility constrained does not trivially bind (Lemma 3). In this case, there is a negative equilibrium collateral spread if and only if the unsecured rate is less than the risk and illiquidity adjusted cost of liquidity in the security cash market when the short borrows up to her capacity in a home-made repo, that is, $u < \hat{y}((1 - \eta)/(1 - \varepsilon_0))$. When this holds, the cost of a home-made repo exceeds the unsecured rate. Thus, the short is willing to pay a repo rate above the unsecured rate. This is equilibrium since the cash provider requires a lower repo rate than the maximum the short is willing to pay, because of the cash provider's larger unsecured borrowing capacity.

Comparing this condition with the condition for a positive equilibrium collateral spread in Theorem 1, we see that positive (negative) collateral spreads are associated with large (small) $u - \bar{y}$. There is a region of overlap, where both negative and positive collateral spreads may occur. The overlap region is

$$(u - \bar{y})/(\rho\sigma_y^2) \in \left[\frac{1 - \kappa}{2}, \frac{1 - \eta}{2} \right), \quad \text{i.e.,} \quad \hat{y} \left(\frac{1 - \kappa}{1 - \varepsilon_0} \right) \leq u < \hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right). \quad (36)$$

In words, both positive and negative collateral spreads are equilibrium if the unsecured rate falls between the risk and illiquidity adjusted costs of liquidity for the short and cash provider when they borrow to their full capacities in their respective cash market problems. To the left of this region, equilibrium collateral spreads are negative. To the right, they are positive.

If the short is less constrained than the cash provider ($\eta > \kappa$), Theorem 2 says that a negative collateral spread is not consistent with equilibrium. One may therefore interpret the fact that negative collateral spreads are common as being consistent with the intuitive idea that cash providers are typically less constrained, or put differently, have "easier" access to liquidity, than cash takers.

The reason a negative collateral spread cannot be equilibrium when $\eta > \kappa$ relates to the fact that it implies that the potential cash provider's unsecured borrowing constraint is binding (Lemma 2). But, as discussed after Theorem 1, when the cash provider faces tighter borrowing constraints and these are binding, the short can do better through a home-made repo than going through the repo market. The maximum rate the short is willing to do a repo at falls below the minimum rate that is acceptable to the constrained

cash provider. Thus, there is no equilibrium repo rate (either positive or negative).

We motivated our paper with reference to the puzzle of negative collateral spreads. Our theory says that this may occur when the unsecured rate drops sufficiently below the expected rate of return of the underlying security. The exact distance depends on risk aversion, volatility, and unsecured borrowing caps. Such a drop may occur as a result of conditions in the unsecured market. It may also occur if securities prices drop so that their expected rates of return rise. In other words, negative collateral spreads may be symptoms of especially high unsecured rates (for example, relating to the reserve maintenance period cycle) or depressed securities prices.

3.4 Remark 1: Role of unsecured borrowing constraints

In our model, there are potentially two riskfree rates, the repo and the unsecured rates, since the setup excludes credit risk by Assumption 2. This does not give rise to arbitrage because of the assumption that both the short (cash taker) and the cash provider are constrained in the unsecured market. If Assumption 1 were dropped, the short could borrow the unit of liquidity she needs at a rate of u , implying that r cannot exceed u . Furthermore, from a cash provider's perspective, it is clear that r could not be less than u . Thus, with no constraints in the unsecured market, we must have $r = u$. The fact that these rates are rarely equivalent in practice suggests that there are constraints in the unsecured market.¹⁶ This makes sense since there is a limited quantity of reserves in the economy. The expressions for the repo rate in Theorems 1 and 2 also show that variations in these constraints can contribute to volatility in the collateral spread.

Our results bear some relation to those in Duffie (1996), although his focus is on special repo rates. In particular, he shows that the special repo rate is at most equal to the riskfree rate (in his notation, $R \leq i$). Duffie's result is driven by trading asymmetries whereby one needs to hold the specific collateral in order to short it. In our model, it is also not possible to sell more than what is owned. However, in Duffie's model, unlike in ours, a player can always borrow at the riskfree rate. This makes the specific security more

¹⁶Repo and unsecured rates could also differ due to differential trading costs.

expensive relative to the bond, resulting in a special repo rate lower than the riskfree rate. Our result is analogous in that $r \leq u$ as long as the unsecured rate is above the cash market adjusted rate of return of the security, which one can think of as the underlying security being relatively “expensive.” However, in our model, the reverse is also possible when the underlying security is relatively “cheap.”

3.5 Predictions and implications

In this subsection, we draw out empirical predictions and implications of the model. We first provide explicit expressions for the collateral spread and present the comparative statics when the spread is positive and when it is negative. In particular, we look at the effects on the collateral spread of the unsecured rate, u ; the (liquidity adjusted) expected rate of return of the underlying security, \bar{y} ; the haircut, h ; volatility, σ_y^2 ; and illiquidity, ε_0 . The effect of risk aversion, ρ , is the same as that of volatility. We then discuss the determinants of the sign of the collateral spread.

Positive collateral spread

Theorem 1 shows that there are three cases when the collateral spread is positive. The first two give rise to a unique repo rate so that comparative statics are easily derived. However, in the third case, the repo rate is indeterminate, bound by \underline{r} and \bar{r} . In this case, we analyze the expressions for $u - \underline{r}$ and $u - \bar{r}$ and present comparative statics for these.

Proposition 1. *If the collateral spread is positive, that is $u - r \geq 0$, expressions for the collateral spread and associated comparative statics are as in Table 4. In particular, the collateral spread is increasing in the unsecured rate, u , and the haircut, h , and decreasing in risk aversion, ρ , and the following properties of the underlying security: the expected rate of return, \bar{y} , volatility, σ_y^2 , and illiquidity, ε_0 .*

Proof: See the Appendix.

The intuition for the comparative statics in Proposition 1 relates to the tradeoff between doing a regular repo and the alternative of a home-made repo. To clarify this, we will focus on the relatively simple case where the short’s feasibility constraint in the home-made repo

Table 4: Collateral spread and comparative statics when $u - r \geq 0$

Case	Collateral spread	Effect of change in					
		u	\bar{y}	h	σ_y^2	ε_0	ρ
1. $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)$	$u - r = \frac{1-\varepsilon_0}{1-h} \left[u - \bar{y} - \frac{1}{2}\rho\sigma_y^2(1 - \varepsilon_0) \right]$	+	-	+	-	-	-
2. $\rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\} \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0)$	$u - r = \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1-h)}$	+	-	+	-	-	-
3. $\eta \leq \kappa$ and $u - \bar{y} < \rho\sigma_y^2(1 - \eta)$							
a. \bar{r} : $u - \bar{y} \geq \frac{1}{2}\rho\sigma_y^2(1 - \eta)$	$u - \bar{r} = \frac{1-\eta}{1-h} \left[u - \bar{y} - \frac{1}{2}\rho\sigma_y^2(1 - \eta) \right]$	+	-	+	-	-	-
b. \underline{r} : $u - \bar{y} \geq \rho\sigma_y^2(1 - \kappa)$	$u - \underline{r} = \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1-h)}$	+	-	+	-	-	-
c. \underline{r} : $\frac{1}{2}\rho\sigma_y^2(1 - \kappa) \leq u - \bar{y} < \rho\sigma_y^2(1 - \kappa)$	$u - \underline{r} = \frac{1-\kappa}{1-h} \left[u - \bar{y} - \frac{1}{2}\rho\sigma_y^2(1 - \kappa) \right]$	+	-	+	-	-	-

Notes. Case 3: If $\eta = \kappa$, $\underline{r} = \bar{r}$. Case 3(a): The condition here ensures $\bar{r} \leq u$.

problem, (7), binds. That is, in the home-made repo, the short optimally chooses to sell the entire holding of her underlying security and, thus, minimize her unsecured borrowings. This is listed as Case 1 in Table 4. In this case, as seen in Theorem 1, there is a unique equilibrium repo rate as given by (18). This can also be written as

$$\begin{aligned} (1-h)r + hu &= (1-\varepsilon_0)\hat{y}(1) + \varepsilon_0 u, \\ \text{where } \hat{y}(1) &= \bar{y} + \frac{\rho}{2}\sigma_y^2(1-\varepsilon_0). \end{aligned} \tag{37}$$

This says that, in equilibrium, the weighted average cost of liquidity from doing a repo equals the weighted average cost from doing a home-made repo. Recall that $\hat{y}(1)$ is the risk and illiquidity adjusted cost of liquidity from selling the entire security in the cash market and repurchasing it at date 1. As an initial observation, note we must have $\hat{y}(1) < u$ because $r < u$ (by assumption).

Given (37), the comparative statics of the collateral spread, $u-r$, are relatively straightforward. First, it is clear from (37) that an increase in the unsecured rate, u , implies that the repo rate must change by less than the unsecured rate in order to keep the costs of the two alternatives equal. Thus, the collateral spread is increasing in the unsecured rate.

Second, if $\hat{y}(1)$ increases, the repo rate must increase relative to the unsecured rate in order to keep the two alternatives equally costly, implying a drop in the collateral spread. In other words, an increase in the security's expected rate of return, \bar{y} , or its volatility, σ_y^2 , cause a decrease in the collateral spread, *ceteris paribus*.

Third, an increase in the haircut, h , increases the cost of liquidity from the repo alternative, since $r < u$. This makes the home-made alternative more attractive. Thus, the repo rate must drop relative to the unsecured rate in order to keep the costs of the two alternatives the same.

Fourth, the effects of an increase in illiquidity, ε_0 , can also be understood intuitively from (37). Note that illiquidity affects both $\hat{y}(1)$ and the weights on the security cash trade and unsecured borrowing in the home-made repo case. As is intuitive, an increase in illiquidity, increases σ_y^2 . Thus, an increase in ε_0 raises the cost of the home-made repo since it also puts more weight on unsecured borrowing. As a result, the repo rate must increase relative to the unsecured rate in order to keep the cost of the regular repo alternative equal

to that of the home-made repo. In short, the collateral spread must tighten.

Equation (37) also emphasizes a key point of our analysis, namely that when agents in the market for liquidity face constraints in the unsecured market, the repo rate, the unsecured rate, and the security cash market expected rate of return must be linked. Case 1 in Table 4 gives a clear-cut link because, in this case, the largest repo rate at which the short is willing to trade coincides with the smallest acceptable rate for the cash provider. Proposition 1 can be thought of as showing that the intuition above carries through when these bounds do not coincide.

Negative collateral spread

As seen in Theorem 2, a negative collateral spread is an equilibrium phenomenon only if the short has a smaller unsecured borrowing capacity than the cash provider, $\eta \leq \kappa$. When this inequality is strict, the repo rate is not uniquely determined, but lies in an interval (R_{neg}, \bar{r}_{neg}) , where $R_{neg} = \max\{u, \underline{r}_{neg}\}$. If $\eta = \kappa$ the upper and lower bounds coincide and define a unique equilibrium $r > u$. As above, we analyze the comparative statics for the collateral spread using the lower and upper bounds of the repo rate when a unique rate does not exist.

Proposition 2. *If the collateral spread is negative, that is $u - r < 0$, expressions for the collateral spread and associated comparative statics are as in Table 5. In particular, the collateral spread is increasing in the unsecured rate, u and decreasing and decreasing in risk aversion, ρ , and the following properties of the underlying security: the liquidity adjusted expected rate of return, \bar{y} , volatility, σ_y^2 , and illiquidity, ε_0 . An increase in the haircut, h , has a nonnegative effect on the collateral spread.*

The comparative statics are, with one exception, the same as for the case of a positive collateral spread. The basic intuition for the similarity is that the fundamental tradeoffs between the regular repo and the home-made repo do not change when collateral spreads are negative. Our focus here is, therefore, on the exception, namely the haircut. In four out of the six scenarios in Table 5, this is seen to have no effect on the collateral spread.

The irrelevance of the haircut can be understood by first noting that when $r > u$, the short borrows as much as possible in the unsecured market under the regular repo

Table 5: Collateral spread and comparative statics when $u - r < 0$

Case	Collateral spread	Effect of change in					
		u	\bar{y}	h	σ_y^2	ε_0	ρ
1. $\eta < \kappa$							
a. \bar{r}_{neg} :	$u - \bar{r}_{neg} = u - \bar{y} - \frac{1}{2}\rho\sigma_y^2(1 - \eta)$	+	-	0	-	-	-
b. \underline{r}_{neg} : $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)\phi$	$u - \underline{r}_{neg} = \frac{1-\varepsilon_0}{1-h} \left[u - \bar{y} - \frac{1}{2}\rho\sigma_y^2(1 - \varepsilon_0)\phi \right]$	+	-	+	-	-	-
c. \underline{r}_{neg} : $\rho\sigma_y^2(1 - \kappa) \leq u - \bar{y} \leq \rho\sigma_y^2(1 - \varepsilon_0)\phi$	$u - \underline{r}_{neg} = \frac{1}{2} \frac{(u-\bar{y})^2}{\rho\sigma_y^2(1-\eta)}$	+	-	0	-	-	-
d. \underline{r}_{neg} : $u - \bar{y} \leq \rho\sigma_y^2(1 - \kappa)$	$u - \underline{r}_{neg} = \frac{1-\kappa}{1-\eta} \left[u - \bar{y} - \frac{1}{2}\rho\sigma_y^2(1 - \kappa) \right]$	+	-	0	-	-	-
2. $\eta = \kappa$	$u - r = u - \bar{y} - \frac{1}{2}\rho\sigma^2(1 - \eta)$	+	-	0	-	-	-

Note: $\phi \equiv (1 - \eta)/(1 - h)$. (ii)

alternative. Therefore, the haircut is not relevant for the short's liquidity cost. However, it affects the quantity that she repos, ϕ as defined in (26). Thus, it affects the cash provider's cost of providing liquidity *if* the cash provider would optimally raise liquidity by selling the entire quantity, ϕ , of the underlying security he gets from the reverse repo. This happens in one of the five cases in Table 5 (Case 1b). More generally, the cash provider's feasibility constraint does not bind and, therefore, the haircut generally does not affect the collateral spread when this is negative.

Corollary

As a corollary of Propositions 1 and 2, we have that the collateral spread is increasing in the risk and illiquidity of the underlying collateral. The implication is that baskets that contain more risky and less liquid securities should on average have a lower collateral spread. This is exactly what Bartolini, Hilton, Sundaresan, and Tonetti (2011) observe in the US data. We also see this in the European data (below).

The sign of the collateral spread

The following is a direct corollary of Theorems 1 and 2.

Proposition 3. *Ceteris paribus, the collateral spread can switch from positive to negative if (i) borrowing constraints tighten (η and κ decrease), (ii) the expected rate of return or volatility of the underlying assets increase, (iii) risk aversion increases, (iv) illiquidity increases, (v) the unsecured rate decreases.*

Proof: This follows directly from the conditions in Theorems 1 and 2 for a positive or negative collateral spread. With respect to the effect of illiquidity, note that \bar{y} and σ_y^2 are functions of ε_0 as seen in (1) and the discussion that follows that equation. \square

The intuition is essentially the same as to the intuitions behind the results in Propositions 1 and 2. The only new element here is the role of the borrowing constraints. Intuitively, when these tighten, more of the underlying security will need to be sold in order to raise the required unit of liquidity. This increases the overall risk taken by the cash taker and provider and, therefore, the cost of raising liquidity. This is tantamount to saying that the repo rate must rise relative to the unsecured rate.

3.6 Remark 2: Linked versus non-linked borrowing capacities

In this subsection we address the robustness of our results to Assumption 6 by changing it to:

Assumption 6'. *If the short draws down on her unsecured borrowing capacity, the unsecured borrowing capacity of the cash provider is unchanged.*

Consider first a positive collateral spread. Assumption 6' does not change that, in the case of Alternative 1, the short optimally repos the entire security she holds. It also does not affect her optimal trade size, Ω , under Alternative 2. So from the short's direct perspective, Assumption 6' is immaterial. However, Assumption 6' changes the borrowing constraint of the cash provider. As already noted in footnote 14, this now becomes $\alpha(1 - \varepsilon) \geq 1 - h - \kappa$. This is because the cash provider's obligation to the short is only to provide $1 - h$ in cash. Intuitively, because the cash provider's funding requirement is h less than that of the short, he can be thought of as effectively increasing his unsecured borrowing capacity by h . If we define $\kappa' \equiv \kappa + h$, it is clear that the analysis for a positive collateral spread under Assumption 6' is identical to the analysis under Assumption 6, but with κ' replacing κ everywhere.

Consider next a negative collateral spread. Again, only the cash provider's borrowing constraint is directly affected. Now this becomes $\alpha(1 - \varepsilon) \geq 1 - \eta - \kappa$. Defining $\kappa'' \equiv \kappa + \eta$, the analysis is the same as under Assumption 6, but with κ'' in place of κ . However, there is no analogue to the scenario $\eta > \kappa$ because η is always strictly less than κ'' . Thus, under Assumption 6', there is equilibrium $r > u$ if and only if $u - \bar{y} < \frac{1}{2}\rho\sigma_y^2(1 - \eta)$, regardless of the relation between η and κ .

The key differences between the analyses under Assumptions 6 and 6', as implied by the above discussion, are summarized in Table 6. We see that, *ceteris paribus*, the range over which an equilibrium repo rate exists expands when borrowing capacities are not linked. One may interpret this as saying that linked unsecured credit lines and limits reduce the ability of repos to serve as an alternative source of liquidity to banks. The range over which there is a positive collateral spread also expands under unlinked borrowing capacities and

Table 6: Positive vs negative equilibrium collateral spreads.

<i>Panel a: Assumption 6. Borrowing capacities are linked.</i>				
	$\eta \leq \kappa$		$\eta > \kappa$	
Spread:	Positive	Negative	Positive	Negative
if $\frac{u-\bar{y}}{\rho\sigma_y^2}$	$\geq \frac{1-\kappa}{2}$	$\leq \frac{1-\eta}{2}$	$\geq 1 - \kappa$	Not equilibrium

<i>Panel b: Assumption 6'. Borrowing capacities are not linked.</i>				
	$\eta \leq \kappa + h$		$\eta > \kappa + h$	
Spread:	Positive	Negative	Positive	Negative
if $\frac{u-\bar{y}}{\rho\sigma_y^2}$	$\geq \frac{1-\kappa-h}{2}$	$\leq \frac{1-\eta}{2}$	$\geq 1 - \kappa - h$	$\leq \frac{1-\eta}{2}$

when haircuts increase. Intuitively, this is because unlinking borrowing capacities and increasing haircuts reduce the effective funding burden of the reverse position.

Examination of the expressions for the collateral spread derived in Propositions 1 and 2 (as presented in Tables 4 and 5) and keeping in mind the above discussion shows that the comparative statics are the same as before except possibly with respect to h in Case 3c in Table 4, because κ should here be replaced by κ' , which is a function of h . However, given the conditions on $u - \bar{y}$ in Case 3c, it is straightforward that the effect of an increase in h remains positive. Thus, the effects of switching from Assumption 6 to Assumption 6' are relatively minor. This is not all that surprising since the basic forces in the model are not affected by this switch.

4 Empirical analysis

In this section, we test some of the key predictions of the model using data on unsecured rates and repo rates from the euro area. With respect to repo rates, we make use of a comprehensive intraday dataset of repo transactions from Eurex Repo, who offers a trading platform for anonymous repo transactions.¹⁷ The sample period runs from January 2, 2007 to June 30, 2015. The dataset contains both general collateral and special con-

¹⁷See www.eurexrepo.com/repo-en/ and www.eurexgroup.com.

tracts. Because our model is based on using repos to raise liquidity, we focus on general collateral repos.¹⁸ For the unsecured rate, we use the Eonia, which is a volume-weighted average of overnight unsecured euro transactions by reporting (European) banks.¹⁹ Abbassi, Bräuning, Fecht, and Peydró (2014) provide evidence that trading in the unsecured market remained active after the start of the financial crisis and the collapse of Lehman Brothers (September 15, 2008), especially at the short end. Our repo data allows us to calculate overnight volume-weighted repo rates. Thus, we calculate collateral spreads by using overnight volume-weighted unsecured and repo rates on a day-by-day basis.

These overnight collateral spreads are then used to test our theoretical model, namely with respect to how the collateral spread reacts to changes in the unsecured rate, haircuts, and volatility. Our tests exploit different institutional elements in the market for liquidity in the euro area. We will also provide further evidence that repos based on lower collateral quality result in larger repo rates in a setting where this cannot be due to credit risk.

4.1 General collateral repos: Contracts and transactions

We start the empirical analysis by describing our general collateral repo data in terms of types of contract and volume by contract type and tenor. Eurex Repo provides two types of general collateral contracts; namely, GC Pooling and Euro Repo. Trading in both GC Pooling and Euro Repo is open to credit institutions and investment firms. All trades are anonymous, cleared by the CCP Eurex Clearing, and settled by Clearstream. Tenors may be overnight, tomorrow-next, spot-next, longer-term, and variable term. We focus on overnight and tomorrow-next. Volume at longer maturities is small and, in any case, not relevant with respect to calculating overnight collateral spreads.

GC Pooling (GCP) includes the ECB and ECB Extended baskets, which will be our focus, as well as a basket containing equities. “Basket” refers to a list of securities (by ISIN) that can be used as collateral. The GCP ECB and GCP ECB Extended baskets are subsets of the 30,000 to 40,000 securities on the public list of eligible collateral in Eurosystem liquidity injecting operations (see Nyborg, 2016). In September 2013, there

¹⁸See Rösler (2017) for an analysis of the special repo transactions.

¹⁹Eonia is an acronym for Euro Overnight Index Average. See <http://www.euribor-rates.eu/eonia.asp>.

were around 7,000 securities in the GCP ECB basket and 20,000 in the GCP ECB Extended basket, ranging from government bonds to unsecured bank bonds with different domiciles. The ECB basket represents higher quality collateral than the ECB Extended basket. For example, securities in the GCP ECB basket need a minimum rating of A− (on the S&P scale), while those in the GCP ECB extended basket need at least a BBB− rating.²⁰ The ECB Extended basket was created after the ECB started admitting collateral rated below A− (down to BBB−) in its operations on October 25, 2008. Thus, the sample start date is later for the ECB Extended basket (November 24, 2008) than the ECB basket (January 2, 2007).

Unlike the GCP baskets, Euro Repo baskets are constrained to one type of security, e.g. French covered bonds. The minimum rating for inclusion is A−. There are two other notable differences between GC Pooling and Euro Repo contracts. First, in a GC Pooling transaction, the cash taker pledges securities from her collateral account; whereas, in Euro Repo transactions, she transfers these securities physically to the counterparty. While re-use in Euro Repo has no limitations, collateral obtained in a GC Pooling transaction can only be used in other GC Pooling transactions, in Eurosystem operations, and, in the case of the GCP ECB basket, in other transactions at the CCP (e.g. in futures contracts). Second, haircuts are different. Eurex’ policy is to derive haircuts for the GC Pooling contracts from haircuts in Eurosystem operations. Nyborg (2016) provides evidence that they are identical in around 90% of cases. Eurex may increase haircuts for paper where it deems risk to be especially large. In contrast, Euro Repo haircuts are set by the clearing counterparty, Eurex Clearing, without an explicit link to Eurosystem haircuts.

[insert Table 7 about here]

Table 7 reports volume and rate statistics for all GC Pooling and Euro Repo baskets, based on all 261,663 general collateral transactions in our dataset. There are 28 different baskets in total. These are ranked in the table by transaction volume per trading day. The GC Pooling ECB and ECB Extended baskets are the most active, with average trading-day

²⁰S&P denotes Standard and Poor’s. All references to ratings in this paper are on the S&P scale unless otherwise specified.

volumes of EUR 40,311 and 18,333 million, respectively. These two baskets account for approximately 84% of all transactions. The majority of these are overnight repos. These two GC Pooling baskets comprise around 99.5% of all overnight repo transactions.

The most active Euro Repo basket is the German KfW/Laender basket, with an average daily volume of EUR 1,645 million. All transactions in this basket have a tomorrow-next tenor, as do most transactions in the Euro Repo space.

Table 7 also reports on volume-weighted average collateral spreads, relative to the Eonia, for all baskets. We use overnight transactions for the GC Pooling ECB and ECB Extended baskets and tomorrow-next transactions for all other baskets. In the former case, we use the Eonia of the day of the transaction, and, in the latter case, we use the next day's Eonia. The volume-weighted average collateral spread across all overnight transactions is 4.07 basis points (bps) for the ECB basket and 1.38 bps for the ECB Extended basket. As these are both CCP repos, it does not seem plausible that this difference relates to credit risk. However, the larger average collateral spread for the higher quality baskets is consistent with the predictions of our model. Overall, the largest (smallest) collateral spread is for the Euro Repo Germany 10 Year basket (Euro Covered bonds), with 15 (-6.14) bps. This is also in line with our model.

[insert Table 8 about here]

In the more formal empirical tests below, we use overnight repo trades only to calculate collateral spreads (since the Eonia is an overnight rate). Thus, our focus is on the two main GC Pooling baskets, since these represent nearly all overnight transactions. Table 8 provides descriptive statistics on the collateral spreads based on overnight trades in these two baskets. The underlying data is the same as what is used in Table 7. However, for Table 8, we first calculate volume-weighted average collateral spreads for each day. The table then reports summary statistics of the resulting sample of these daily averages. This is also what is plotted in Figure 1.

Both GCP collateral spreads are positive on average, but lower for the lower quality basket. The Eonia – GCP ECB (Extended) basket has a mean of 3.773 (1.362) bps. Both are significant at the 1% level and significantly different from each other. The median is

positive for both: 4.109 bps and 2.371 bps, respectively. The collateral spread calculated using the ECB Extended basket is lower than that based on the ECB basket on 97.506% of the 1,604 overlapping days. For the ECB basket, the collateral spread is negative on 22.889% of all days; whereas for the ECB basket, it is negative 28.741% of the time. These basic findings support the prediction of our theory that collateral spreads are increasing in the quality of the underlying collateral. Our findings on this point mirror those of Bartolini, Hilton, Sundaresan, and Tonetti (2011) using US data. Credit risk is not a plausible explanation in our case because we are using CCP repos and overnight trades.

4.2 Empirical test: Spikes

In this subsection, we test our model by exploiting the feature of money markets that unsecured rates tend to spike, up or down, toward the end of the reserve maintenance period. Since longer-term rates and securities market do not experience these spikes, this allows us to test the prediction in Propositions 1 and 2 that the collateral spread is increasing in the unsecured rate. That is, the collateral spread is predicted to spike in the same direction as the unsecured rate.

We carry out the “spikes test” on the subset of the sample period when the ECB operated with a liquidity-neutral policy (Nyborg, Bindseil, and Strebulaev, 2002).²¹ That is, it aimed to inject the quantity of central bank money (liquidity) that banks needed to fulfill reserve requirements and satisfy other liquidity needs *in aggregate* within each (approximately monthly) maintenance period. At the end of the maintenance period, an excess of liquidity in the system would drive the overnight rate down toward that of the deposit facility, 100 bps below the policy rate. A liquidity shortage would drive the overnight rate up toward the marginal lending facility (“discount window”) rate, 100 bps above the policy rate. Thus, equal likelihood of an excess or a shortage should then, in theory, keep the overnight rate close to the policy rate before the end of the maintenance period. As documented, for example, by Perez-Quiros and Mendizabal (2006), Nautz and

²¹For descriptions of the basic Eurosystem monetary policy framework see European Central Bank (2002) or European Central Bank (2011).

Offermanns (2007), Linzert and Schmidt (2011), the ECB was successful in controlling the overnight rate this way. The relevant takeaway for this paper is that spikes were built in to the operational framework of the ECB under the liquidity-neutral policy. This policy stayed in place until October 2008, when, in the aftermath of Lehman’s bankruptcy, the ECB introduced its full allotment policy. We run our spikes test over the period January 2, 2007, to August 29, 2008, with the end-date representing the end of the last month before Lehman’s default.²²

End-of-calendar-month spikes in the Eonia have also been documented in the literature, which may be related to window dressing by banks (Bindseil, Weller, and Wuertz, 2003; Fecht, Nyborg, and Rocholl, 2008). This is also considered in the tests below. Unlike end-of-maintenance period spikes, which may be up or down, end-of-month spikes are (nearly) uniformly up. They are also somewhat weaker.

Eonia: End-of-maintenance-period and end-of-month spikes

As a first step in our spikes test, we first verify spikes in the Eonia over the sample period. We do this by calculating abnormal, or standardized, levels of the Eonia at the end of each maintenance period and calendar month. We then look at the average standardized end-of-maintenance-period and end-of-calendar-month Eonia over the sample period, breaking the data out into positive and negative end-of-period changes. We also look at the ranks of days within maintenance periods based on the Eonia.

We proceed as follows: First, for each maintenance period, m , we calculate the mean, \bar{u}_m , and standard deviation, s_m , of the Eonia, excluding the last five days and the last trading day of the calendar month. For each day, t , within the maintenance period, we then calculate the standardized Eonia as

$$\text{stand. Eonia}_{t,m} = \frac{u_{t,m} - \bar{u}_m}{s_m}. \quad (38)$$

[insert Table 9 about here]

²²This represents twenty-one reserve maintenance periods, with the first maintenance period starting on December 13, 2006 and the last ending on September 9, 2008. So we have the end-periods of twenty reserve maintenance periods, with the twentieth maintenance period ending on August 12, 2008.

Table 9 reports on the standardized Eonia, across maintenance periods, for the last five days of the maintenance period, labeled endmp (last day) to endmp-4 (four days before the last day), and the last day of each calendar month, labeled endmonth. The average for endmp is -7.675. In other words, the Eonia is, on average, 7.675 standard deviations lower on the last day of the maintenance period than its normal, within-maintenance period, level. Thus, the Eonia typically spikes down at the end of the maintenance period. This overall downward move is also seen in the days immediately before the last day, with the standardized Eonia ranging from -3.585 (endmp-1) to -1.474 (endmp-2) on average. These large negative averages mask the fact that the Eonia also may spike up. The table shows that there are nine up-spikes on the last day of the maintenance period, with five of these representing spikes of more than two standard deviations. There are eleven down-moves, with seven of these representing spikes of more than two standard deviations. The absolute value of the standardized Eonia on the last day of the maintenance period averages to 16.310 across maintenance periods. In short, there are both statistically significant up- and down-spikes in the Eonia at the end of the maintenance period over the sample period.

With respect to the end of the month, the last row in Table 9 shows that there are nineteen end-of-month up-moves, with only one small down-move. Thirteen of the up-moves represent standardized Eonias of more than two standard deviations. The average end-of-month standardized Eonia is 3.583. Thus, the Eonia tends to spike up at the end of the month.

To further examine end-of-maintenance period spikes in the Eonia, we rank the standardized Eonia within each maintenance period, focusing on the last five days. The order of the ranking is determined by the standardized Eonia on the last day of the maintenance period. If this is negative (positive), the ranking is in ascending (descending) order. So if the standardized Eonia on the last day of the maintenance period is negative (positive) a rank of one is given to the day with the lowest (highest) standardized Eonia. A rank of two is given to the day with the second lowest (highest) standardized Eonia, and so on. For each maintenance period, Table 10 reports the rankings of the five last days.

We also rank the last day of the month within the maintenance period. Since the standardized Eonia (with one exception) is positive at the end of each calendar month,

this is always done in descending order.

[insert Table 10 about here]

The ranking results are in Table 10. The five last days of the maintenance period contains the first rank, i.e. the day with the largest standardized Eonia (either positive or negative), in fifteen out of the twenty maintenance periods. The last day has the first rank in ten maintenance periods. The last day of the calendar month has the first positive rank (within the maintenance period) in ten maintenance periods and has a rank of three or better in fifteen cases. This is further evidence of spikes in the Eonia at the end of the maintenance period and up-spikes at the end of the calendar month.

Collateral spread spikes test

Based on Propositions 1 and 2, we expect end-of-maintenance-period down- and up-spikes in the Eonia to have a negative and positive effect, respectively, on the collateral spread. This is so because these spikes are not driven by general market conditions, but the operational framework of the ECB. Under the liquidity-neutral policy, spikes at the end of the maintenance period are driven by the central bank's injections of reserves relative to aggregate needs and, to some extent, also by allocational imperfections in the interbank market for liquidity (Bindseil, Nyborg, and Strebulaev, 2009). Thus, end-of-maintenance period spikes provide us with an ideal opportunity to test a central implication of our theory.

As a first step, we identify days with spikes in the Eonia by the 10th and 90th percentiles of the Eonia in each maintenance period. Dummy variables to capture these days are labeled `perc10` and `perc90`, respectively.²³ Second, we condition these spikes on whether they occurred in one of the last five days of the maintenance period or not. Thus, we generate the variables `perc10|endres`, `perc90|endres`, `perc10|nonendres`, and `perc90|nonendres` to capture down and up spikes, respectively, for the last five days and for other days, respectively.

²³The last day of the month is not included when determining `perc10` and `perc90`.

We run two regressions. The first specification is:

$$y_t = \beta_0 + \beta_1 \text{perc10}_t + \beta_2 \text{perc90}_t + \beta_3 \text{fincrisis}_t + \beta_6 \text{monthend}_t + \varepsilon_t, \quad (39)$$

where y_t is the collateral spread, fincrisis is an indicator variable for the financial crisis (days after August 07, 2007, inclusive), and monthend indicates the last trading day of the calendar month. The collateral spread on day t is the Eonia less the volume-weighted average GC Pooling ECB basket rate (for overnight repos only).

The second specification is:

$$y_t = \beta_0 + \beta_1 \text{perc10|nonendres}_t + \beta_2 \text{perc90|nonendres}_t + \beta_3 \text{perc10|endres}_t + \beta_4 \text{perc90|endres}_t + \beta_5 \text{fincrisis}_t + \beta_6 \text{monthend}_t + \varepsilon_t. \quad (40)$$

We base inference on Newey-West standard errors with the number of lags equal to the fourth root of the number of observations, as recommended by Greene (2008). Here, this means five lags.

[insert Table 11 about here]

Table 11 displays the results. In the first specification, the coefficient on perc10 is negative and statistically significant at the 5% level. The coefficient on perc90 is positive and statistically significant at the 10% level. This is consistent with our theoretical predictions. The results for Specification 2, show that it is especially spikes at the end of the maintenance period that matter. As predicted by our theory, the coefficients on perc10|endres and perc90|endres are negative and positive, respectively, and statistically significant at the 5% and 1% levels, respectively. Spikes outside the end of the maintenance period do not seem to matter. This is highly supportive of our theory because the spikes at the end of the maintenance period are driven by the operational framework of the ECB, that is, by a dominating factor with respect to the general level of unsecured rates. The coefficients on monthend are insignificant in both regressions, suggesting that the factors that drive these spikes are not unique to the unsecured market.

Finally, the coefficient on fincrisis is negative and statistically significant at the 1% level in both specifications. This also relates to our model. The financial crisis was a period of

reduced liquidity and depressed security prices. As seen in Propositions 1 and 2, both of these effects would be expected to lead to a reduction in the collateral spread. Thus, our theory is consistent with the negative coefficient on the financial crisis indicator variable. Overall, the spikes tests support our theory.

4.3 Empirical test: Change in haircuts

In this subsection, we build a test around an exogenous change to haircuts based on an institutional feature of Eurex' GC Pooling ECB and ECB Extended contracts. For these two baskets, Eurex bases its haircuts on those used by the Eurosystem in its (reverse repo) operations.²⁴ Moreover, as documented by Nyborg (2016), the ECB has historically updated haircuts only every three to four years. Thus, Eurosystem haircuts do not reflect current developments in money markets. A change to Eurosystem haircuts, therefore, provides an ideal natural experiment for studying the effect of a change in haircuts on the collateral spread. As seen in Proposition 1, when the collateral spread is positive, an increase in haircuts is predicted to increase the collateral spread.

During our sample period, the Eurosystem changed haircuts on marketable securities three times. We use the third haircut adjustment in our test as it occurred during a relatively calm period and involved wide-ranging cuts to haircuts. This was announced on September 27, 2013 and implemented on October 1, 2013.²⁵

The Eurosystem's collateral framework sets haircuts based on the type of security, time to maturity, and rating.²⁶ For the purpose of determining the haircut, marketable securities are classified into five "liquidity categories," briefly summarized as: (I) central

²⁴This is as reported by Eurex and is confirmed by Nyborg (2016), Table 5.6.

²⁵The first Eurosystem haircut adjustment occurred on October 25, 2008, shortly after Lehman's default. This had little effect on the GCP ECB basket since, as documented by Nyborg (2016), haircuts for A– or better rated securities did not change, with the exception of unsecured bonds, which are relatively unimportant for the ECB basket (see Table 13). The ECB Extended basket had not been created yet. The second haircut adjustment occurred on January 1, 2011. This was during a stressed time for the euro area. Moreover, as seen in Nyborg (2016), there was only a limited change in haircuts (for lower quality collateral).

²⁶An exact description of this framework and its evolution over time is provided by Nyborg (2016).

government debt instruments, (II) local and regional government debt instruments as well Jumbo covered bank bonds and supranational/agency bonds, (III) corporate bonds and and non-Jumbo covered bonds, (IV) unsecured bank bonds, and (V) asset-backed securities (ABS's). Haircuts increase in the liquidity category, *ceteris paribus*.

The changes to Eurosystem haircuts for marketable securities on October 1, 2013 are displayed in Table 12. This is computed from Tables 5.3 and Tables 5.4 in Nyborg (2016). We see that for eligible securities with a rating of at least A−, haircuts predominantly fall. The most notable exception is for liquidity category IV, where they do not change. There are also two cases with small increases. For securities with a rating in the BBB− to BBB+ range, Eurosystem haircuts also tend to fall, except for liquidity categories I and II, where they rise.

[insert Tables 12, 13, and 14 about here]

To examine the relevance of these Eurosystem haircut changes to the two GC Pooling baskets, Tables 12 and 13 show the distributions of securities in the GC Pooling ECB and ECB Extended baskets, respectively, across the five liquidity categories on September 26, 2013, one day before the announced change. As seen, the two baskets do not contain any securities in category V (ABS's). The GC Pooling ECB basket only contains securities with a rating of at least A−, with 96.5% of all securities being in liquidity categories I to III. Thus, Table 11 confirms that Eurosystem haircuts fall for securities in the GC Pooling ECB basket.

For the GCP ECB Extended basket, Table 13 shows that 94.1% of all securities are rated A− or higher, thus falling in the higher rating category. Thus, Eurosystem haircuts predominantly fall for securities in the Extended basket also, but with some exceptions for lower-rated collateral.

The relevance of changes to Eurosystem haircuts derives from the institutional feature that Eurex Repo sets haircuts for securities in the GCP ECB and ECB Extended baskets equal to those in Eurosystem repos (with some exceptions). To examine this more carefully for the October 1, 2013 Eurosystem haircut adjustments, we have obtained GC Pooling haircuts on a daily basis from Eurex Repo over the period September 16 to October 15,

2013. Using this data, we calculate the daily average haircut for each basket. The resulting time series of average haircuts for both baskets are plotted in Figure 2. The change in haircuts is evident. However, it takes place on October 2, 2013 rather than October 1, 2013. On this date, haircuts in the GCP ECB basket fell by 84 bps on average. In the GCP ECB Extended basket, they fell by 49 bps. Assuming collateral spreads are positive, our theory says that this should be associated with a drop in collateral spreads.

[insert Figures 2, 3, and 4]

The development of the Eonia and GC Pooling overnight repo rates can be seen in Figure 3, with the associated collateral spreads in Figure 4. These figures show that collateral spreads were positive prior to the haircut change, except for an end-of-month dip for the GCP ECB Extended basket. Consistent with our theory, Figure 4 shows that collateral spreads were lower after the drop in haircuts on October 2, 2013.

To formally test the effect of the haircut change, we run the following regression:

$$y_t = \beta_0 + \beta_1 \text{newhaircut}_t + \beta_2 \text{endmp1}_t + \beta_3 \text{endmp2}_t + \varepsilon_t, \quad (41)$$

where y_t denotes the collateral spread, based on either the GCP ECB or GCP ECB Extended baskets.²⁷ The variable of interest, *newhaircut*, is a dummy variable that is equal to one starting October 2, when the new haircuts are applied by Eurex Repo. We control for the last day of each of the two maintenance periods covered by the test periods, September 10, 2013 and October 8, 2013, with two separate indicator variables, *endmp1* and *endmp2*, respectively. The regression is run for two-, three-, and four-week periods around October 2, 2013, with the following days removed: the announcement day, September 27, 2013, the last trading day of September, the first trading day of October, and the last trading day of August. Inference is based on Newey-West standard errors with two lags.

[insert Table 15 about here]

Table 15 shows the results. Consistent with our model's prediction, the coefficient on *newhaircut* is statistically significantly negative for both collateral spreads and for all three

²⁷As always, we take the Eonia less the volume-weighted overnight repo rates for either basket.

test periods. For example, for the test period where we use two weeks either side of October 2, 2013, the collateral spread based on the GCP ECB basket falls by 1.461 bps (statistically significant at the 1% level), and the collateral spread for the GCP ECB Extended basket falls by 1.289 bps (statistically significant at the 5% level). When using four weeks around October 2, the coefficients are 1.005 and 1.097, respectively (both statistically significant at the 5% level). In short, consistent with our theory, in the data, an exogenous decrease in haircuts results in lower collateral spreads.

4.4 Empirical test: Volatility

In this subsection, we test the prediction of our model that the collateral spread is decreasing in volatility, σ_y^2 . Intuitively, a higher volatility translates into a larger risk and illiquidity adjusted return $\hat{y}(\cdot)$ of the underlying security, which makes raising liquidity through trading in the security cash market less attractive for the short. Put differently, the cost of liquidity in the home-made repo alternative becomes more expensive. This leads the short to be willing to accept a higher repo rate and puts downward pressure on the collateral spread. To test the prediction, we use the overnight collateral spread based on the GC Pooling ECB and ECB Extended baskets over their respective sample periods.

The basic idea of our test relates to the general finding by Brenner, Pasquariello, and Subrahmanyam (2009) that return volatility increases before the announcement of macroeconomic news. We focus on the central bank's policy rates, which directly affects the pricing of fixed income securities, such as what are in the two GC Pooling baskets. In the euro area, decisions on monetary policy are made by the Governing Council of the ECB, which meets twice within each maintenance period. Interest rates and the monetary policy stance are subject to change in the second of these meetings.²⁸ Interest rate uncertainty is elevated before these meetings. It drops back after the meetings as uncertainty is realized. In turn, this should translate into a reduction in security volatility. Thus, our theory

²⁸The dates of the Governing Council are determined by the ECB one year in advance. Policy rates include the minimum bid rate in the Eurosystem's main refinancing operations (MROs) (under the liquidity-neutral policy) and the fixed tender rate (under the full-allotment policy), the marginal lending facility rate, and the deposit rate. The standard rate change, if any, is 25 basis points.

predicts that collateral spreads fall after the announcement of the monetary policy stance.

To examine this, we run four models using daily changes in the collateral spread as the dependent variable. The four models are all variations of the following general regression specification:

$$\begin{aligned} \Delta y_t = & \beta_0 + \beta_1 \text{govcouncil_mp}_t + \beta_2 \text{govcouncil_nonmp}_t + \beta_3 \text{vstoxx}_{t-1} \\ & + \beta_4 \text{excessliq}_{t-1} + \Gamma'_1 \mathbf{X}_t + \Gamma'_2 \mathbf{Z}_t + \varepsilon_t. \end{aligned} \quad (42)$$

The dependent variable, Δy_t , is the change in the overnight collateral spread from day $t-1$ to day t based on the GCP ECB basket or the GCP ECB Extended basket, respectively $\Delta \text{Collspread}_1$ and $\Delta \text{Collspread}_2$. We use Newey-West standard errors, with the number of lags equal to the fourth root of the number of observations, as recommended by Greene (2008). See Table 16, which reports on the results, for details.

The main right-hand side variables in (42) are `govcouncil_mp` and `govcouncil_nonmp`. These are indicator variables for Governing Council meeting days where the policy rates are subject to change and not subject to change, respectively. Given that volatility should be affected only in the former case, our theory would say that the coefficient on `govcouncil_mp` should be positive, whereas that on `govcouncil_nonmp` should be insignificant.

The regression specification includes three sets of controls. First, all specifications control for market liquidity through the `vstoxx` and excess liquidity injections into the banking system as reported by the ECB, `excessliq`.²⁹ Excess liquidity is measured as the sum of aggregate volumes at the Eurosystem deposit facility and current accounts less volumes at the lending facility and reserve requirements.

Second, we include a vector of control variables, \mathbf{X} , that are used in all four specifications, but not reported on in the results in Table 16. The individual variables are: `perc10|endres`, `perc90|endres`, `fincrisis`, `monthend` (see Subsection 4.2 for definitions), and an indicator variable for the first trading day of each calendar month.³⁰

²⁹`vstoxx` is the implied volatility from options (30 days constant maturity) on the Euro Stoxx 50 (analogous to the VIX in the US), see <https://www.stoxx.com/index-details?symbol=V2TX>, and is downloaded from Bloomberg. Excess liquidity, `excessliq`, is taken from the ECB's webpage, see https://www.ecb.europa.eu/stats/policy_and_exchange_rates/minimum_reserves/html/index.en.html.

³⁰The indicator variable `fincrisis` is 1 from August 7, 2007 to the end of the sample period.

Third, we include another vector of control variables, \mathbf{Z} , in all specifications except the first one (which serves as a baseline). These capture various dates that relate to the ECB's unconventional monetary policies. This is motivated by Szczerbowicz (2015), who argues that there were strong reactions to these policies in the unsecured and repo markets. Following her approach, we include indicator variables for: the settlement days of the first one-year longer term refinancing operation (LTRO, `1yearltro`) on June 25, 2009, and both three-year LTROs (`3yearltros`) on December 22, 2011 and March 1, 2012; the introduction of a zero deposit facility rate (`zerorate`) on July 11, 2012; and full allotment announcements on October 9, 13, and 15, 2008 (`fullallot`).³¹ These are all reported on in the output in Table 16. The `fullallot` dummy is not used in the `ΔCollspread_2` regressions, since the sample period for the GCP ECB Extended basket starts at a later date. \mathbf{Z} also includes several other key dates relating to ECB unconventional monetary policies which are suppressed in the output.³²

The third and fourth specifications refine the analysis in the second specification by modifying the underlying data in two ways. Specifically, in the third specification we seek to isolate the effect of potential interest changes on volatility from the effect on expected rates of return of the underlying collateral. A change in the interest rate, if unanticipated, changes the expected rate of return of the underlying collateral and, by our theory, also the collateral spread unless the unsecured rate changes by the same amount. Brand, Buncic, and Turunen (2010) show that short-term news by the ECB, such as interest rate decisions, have a significant impact on the yield curve, especially at the short end. So in order to

³¹Full allotment for MROs was announced on October 8, 2008. However, Szczerbowicz (2015) reports that this was in the evening, which is why we dummy the following day. On October 13, 2008, the ECB announced full allotment USD injecting operations (in cooperation with the Fed). On October 15, full allotment was announced for the LTROs.

³²The additional indicator variable elements of \mathbf{Z} , which are not reported on in the regression results in Table 16, are: (i) news on EFSF/ESM on May 10, 2010, March 14, 2011, and March 26, 2011; (ii) announcements of covered bond purchase programmes on May 7, 2009, October 6, 2011, and September 4, 2014; (iii) announcements on one-year and three-year LTROs on May 7, 2009, and December 8, 2012. (iv) the announcement of the public sector purchase programme on January 22, 2015. For discussions of these events, see Szczerbowicz (2015) and Nyborg (2016).

capture the volatility effect more cleanly, in the third specification, we exclude Governing Council meeting days when the main policy rate is actually changed.³³

In addition, in the fourth model, we account for the fact that monetary policy decisions by the Governing Council are announced in the middle of the day, namely through a press release posted on the ECB's webpage at 13:45 CET. Thus, when calculating the daily change in the collateral spread on Governing Council meeting days when interest rates are subject to change, we only use repo transactions with timestamps after 13:45:00 to calculate collateral spreads.

[insert Table 16 about here]

Table 16 shows the results. We first discuss the collateral spread based on the higher quality basket, GCP ECB.

In our baseline specification (Model 1), the coefficient on our main variable of interest, `govcouncil_mp`, is 0.640 (statistically significant at the 5% level). So, on average, the collateral spread increases by approximately 0.640 bps on days when policy rates are subject to change. This is around 16% of the average collateral spread of 3.773 bps over the sample period and 9.5% of the sample standard deviation (see Table 8). In contrast, the coefficient on `govcouncil_nonmp` is not statistically significantly different from zero. So there is a systematic relationship between Governing Council days and the collateral spread only when the policy rates are subject to change and, therefore, volatility is affected. The positive coefficient on `govcouncil_mp` and the insignificant coefficient `govcouncil_nonmp` are consistent with the predictions of our theory.

Our baseline findings hold up in Model 2, which includes a larger set of controls, as described above. The coefficients on `govcouncil_mp` and `govcouncil_nonmp` and their standard errors are almost identical to what they are in Model 1. The results in Model 3 are also very similar. This speaks to the robustness of the findings in Model 1 and the relevance of our theoretical model.

³³The main policy rate is the minimum bid rate in the MROs over the liquidity-neutral period and the tender rate in the full-allotment period.

The results are even stronger in Model 4, where we restrict ourselves to transactions timestamped after 13:45:00 CET on Governing Council days where interest rates are subject to review.³⁴ The coefficient on `govcouncil_mp` shoots up to 2.168 bps, nearly three times as large as before and almost 60% of the average in-sample collateral spread. In a test not reported in the table, we have used pre-13:45 transactions instead. In this case, the coefficient is statistically insignificant, which supports the view that the announcement of the interest rate decision matters in that it resolves uncertainty and reduces volatility. The coefficient on `govcouncil_nonmp` remains insignificant in Model 4. So, overall, the results using the collateral spread based on overnight trades in the GCP ECB basket are solidly consistent with the prediction of our theory that a decrease in volatility leads to a higher collateral spread.

As a counterpoint to this, the coefficient on `govcouncil_mp` is not significant in any of the regressions involving the collateral spread based on the GCP ECB Extended basket. A potential explanation can be obtained from our formulas of the collateral spread in Tables 4 and 5. It may well be that the securities in this basket are so risky and illiquid that relatively small changes in the central bank's policy rate do not matter much. As seen in Table 14, more than half of all securities in the GCP ECB Extended basket are unsecured bank bonds.

As a final remark to the results in Table 16, we also see that an increase in excess liquidity in the banking system reduces the collateral spread for either basket. This relates to the predictions of our theory that collateral spreads are decreasing in the illiquidity of the underlying collateral and the unsecured borrowing capacities of banks. Our finding is consistent with the view that excess liquidity uptakes by banks are a response to heightened illiquidity in the market. Overall, the findings in this subsection provide further support for the relevance of our theoretical model and its predictions.

³⁴The number of observations drops in Model 4 because there are some Governing Council meeting days with no trades after 13:45 CET.

5 Evolution of collateral spreads

Having established that our theory holds up to empirical tests, we next employ it to analyze the development of the collateral spreads in Figures 1a and 1b over time, i.e., from January 2, 2007 to June 30, 2015. For either GC Pooling basket, the figures show regular spikes in the collateral spread. As studied in Subsection 4.2, this relates to end-of-maintenance period spikes in the unsecured rate. After the introduction of the full allotment policy, the spikes are predominantly downward. This is because the full allotment policy was associated with an excess quantity of liquidity in the system. More generally, ignoring the spikes, it can be seen that collateral spreads are negative and positive over extended periods of time. Our focus here is on discussing the broad, non-spike related evolution of the collateral spreads in Figures 1a and 1b.

As indicated by vertical bars in the two figures, several important events occurred throughout the sample period that may have affected conditions in the market and, therefore, collateral spreads. We use the events, which are linked to the financial crisis and the ECB's unconventional monetary policies, to split the time period into eight subperiods as follows:

1. Pre-crisis, Jan 2, 2007 – Jul 31, 2007.
2. Early crisis, Aug 1, 2007 – Oct 8, 2008.
3. Start of full allotment, Oct 9, 2008 – Jun 24 2009.
4. First one-year LTRO by the ECB, Jun 25, 2009 – Jul 1, 2010.
5. Growing euro area sovereign crisis, Jul 2, 2010 – Dec 21, 2011.
6. Three-year LTROs, Dec 22, 2011 – Jun 30, 2013.
7. One-third of LTROs repaid early, Jul 1, 2013 – Jan 21, 2015.
8. The period after the announcement of the Public Sector Purchase Programme (PSPP) by the ECB on Jan 22, 2015. This constitutes the largest programme within the ECB's Quantitative Easing programme.³⁵

³⁵The LTROs originally had terms of around three months. During the course of the financial crisis, the Eurosystem increased the tenor of several LTROs to one and three years to combat the crisis.

Table 17 reports descriptive statistics on the two GC Pooling basket overnight collateral spreads over these eight subperiods. Note that the spread based on the GC Pooling ECB Extended basket is only available from the third subperiod, since this basket was only created after the implementation of the full-allotment policy.

[insert Table 17 about here]

5.1 Negative collateral spread, Eonia–GC Pooling ECB basket

As seen in Table 17, the spread based on the GC Pooling ECB basket is predominantly negative only in Subperiod 2. This subperiod represents the early part of the crisis, ending with the introduction of full allotment shortly after Lehman’s bankruptcy. Ignoring spikes, the collateral spread initially turned negative in the Spring of 2008. This was a time when sovereign yields in the euro area started to diverge (Cline, 2014; Nyborg, 2016). In terms of our model, this can be thought of as \bar{y} and, to some extent, σ_y^2 , starting to rise. Propositions 1, 2, and 3 tell us that a rise in these parameters could contribute to collateral spreads turning negative.

The spread is especially large negative immediately before full allotment – around the time of Lehman’s default. This may reflect two factors. First, securities prices took a large hit around this time. Second, liquidity started to “dry up” in the interbank market. We know from Proposition 3 that tighter borrowing constraints in the unsecured markets may cause collateral spreads to go negative. A combination of depressed security prices and tight borrowing constraints would be our interpretation of the strong negative collateral spread seen in 2008.

The introduction of full allotment on October 25, 2008, sees the collateral spread immediately turn positive. This can also be understood in terms of our theory. First, by providing, in principle, limitless quantities of liquidity, the central bank may have loosened up borrowing constraints in the interbank market. Second, this may also have helped in terms of raising security prices, reducing volatility, and improving liquidity in the security cash market, all of which have a positive effect on the collateral spread according to our theory.

Further developments in the collateral spread based on the GC Pooling ECB basket are discussed below, together with the GC Pooling ECB Extended basket.

5.2 Both collateral spreads

The collateral spread based on the Extended basket is negative, on average, in Subperiods 3 and 5. These negative periods ended with the introductions of the first one-year LTRO and first three-year LTRO, respectively. These operations injected substantial liquidity into the banking system (around EUR 500 billion each) and put upward pressure on security prices. Woschitz (2017) provides evidence that yields fell with the introduction of the first three-year LTRO. Thus, consistent with our model, collateral spreads rose. For the ECB Extended basket they turned positive again, and for the ECB basket they also increased.

As seen in Figure 1, the positive effect of full allotment and one- and three-year LTROs eventually subsides. Our broad interpretation of this is that these policies were not able to permanently fix the underlying problem of illiquidity and relative downward pressure on the large set of securities that make up the two GCP baskets. However, both collateral spreads increase again with the introduction of the Eurosystem's public sector purchase programme (Subperiod 8). This followed a period of falling collateral spreads associated with the prepayment of one-third of the approximately EUR 1 trillion of liquidity provided in the two three-year LTROs. The reduction and subsequent increase in collateral spreads can, once again, be explained through monetary policy that adds liquidity to the system, puts upward pressure on security prices (and potentially also reduces volatility). This perspective is what comes out of our model. In short, our theory is well capable of explaining both the spikes and the broad patterns in collateral spreads over time.

Developments in the collateral spread, which at first may seem counterintuitive, make sense when using our theory. Fluctuating conditions in the unsecured market and the security cash market cause the collateral spread to vary over time. It turns negative if the unsecured rate drops below the adjusted rate of return in the securities market. This may result from conditions in the unsecured market. It may also occur if securities prices drop so that their adjusted rate of return rise. The collateral spread is more prone to turn

negative if conditions in the unsecured markets are especially tight. In this section, we have emphasized that central bank policies may influence the collateral spread by affecting security markets and the availability of liquidity in the system.

6 Concluding remarks

We motivated this paper by the puzzle of negative collateral spreads, that is, repo rates that are larger than unsecured rates. As documented in this paper, this is a regular phenomenon in the euro overnight market. It is also common in the US. This is at odds with the dominant paradigm of credit risk as the main driver of interest rate spreads in the market for liquidity. It suggests that other forces are at work. Credit risk also cannot explain our finding that CCP repo rates based on lower-quality collateral are larger than those based on higher-quality collateral, since the CCP assumes the counterparty risk. In this paper, we have developed a theory of repo rates that is capable of explaining these findings as well as overall trends in the collateral spread over time.

Our fundamental contribution is deriving a constrained-arbitrage relation between the repo rate, the unsecured rate, and the expected rate of return of the underlying asset. Put differently, we show that liquidity constraints imply that these three rates are jointly determined. As an approximation, negative collateral spreads occur when the risk and illiquidity adjusted rate of return of the underlying security exceeds the unsecured rate. So negative collateral spreads are symptoms of low unsecured rates, for example due to reserve maintenance period effects, or depressed and illiquid security markets. Capacity constraints in the unsecured market limits the resulting arbitrage opportunity and allow negative spreads in equilibrium. Such constraints are consistent with the fact that at any point in time there is a fixed quantity of liquidity in the system. Furthermore, there is evidence that the market for liquidity is allocationally inefficient (Bindseil, Nyborg, and Strebulaev, 2009).

The theory in this paper is based on the simple observation that the alternative to raising liquidity through a repo is to sell the underlying security in the cash market and buy it back later. We refer to this as a home-made repo. The specific setup we analyze

assumes that the unsecured market is needed to raise the requisite quantity of liquidity regardless of which alternative is used, regular or home-made repo. This can be thought of as a collateral constraint. Furthermore, the potential cash provider in a regular repo is also liquidity constrained so that he needs to sell a portion of the underlying security to finance the reverse repo. This cements the constrained-arbitrage relation between the three rates in the model. Different capacity constraints for the cash taker and provider mean that there may be a range of equilibrium repo rates. The collateral spread can go from positive to negative when unsecured borrowing constraints tighten as this increases the overall risk of the transaction, since a larger portion of the underlying security will have to be sold to generate the quantity of liquidity that the short player needs.

Our theory yields a number of other implications and predictions. For example: (i) Negative collateral spreads require the cash provider to be less constrained in the unsecured market than the cash taker. (ii) The collateral spread is decreasing in the following properties of the underlying security: the expected rate of return, the volatility, and illiquidity (pricing errors). (iii) It is also decreasing in risk aversion. (iv) The collateral spread is increasing the unsecured rate. (v) If the collateral spread is positive (negative) it is increasing (non-decreasing) in the haircut.

In the second half of the paper, we test some of these predictions using a comprehensive transactions dataset from Eurex Repo. We calculate overnight collateral spreads relative to the Eonia based on the two most active baskets in the overnight market, namely the GCP ECB basket and the GCP ECB Extended basket. In designing our tests, we make use of certain institutional features of these contracts and monetary policy implementation in the euro area. In particular, we exploit spikes in the unsecured rate due to the reserve maintenance cycle, an exogenous haircut change due to the fact that Eurex uses Eurosystem haircuts for the two GCP ECB baskets, and the heightened volatility on Governing Council days when the policy rate is subject to change. All tests strongly support the predictions of the model.

To the best of our knowledge, this is the first paper to highlight the puzzle of negative collateral spreads and, more generally, develop a theory to explain the determinants of the collateral spread. The fundamental insight of our theory is that the repo rate, the unse-

cured rate, and the illiquidity and risk-adjusted expected rate of return of the underlying security are linked. Given unsecured and repo rates, the expressions in this paper may be used to estimate expected rates of return of securities. In more broad terms, our theory provides a fresh perspective on the view that liquidity drives security prices and may serve as a useful starting point for further research into that idea.

References

- Abbassi, P., Bräuning, F., Fecht, F., Peydró, J.-L., 2014. Cross-border liquidity, relationships and monetary policy: Evidence from the euro area interbank crisis. Deutsche Bundesbank Discussion Papers 45.
- Afonso, G., Kovner, A., Schoar, A., 2011. Stressed, not frozen: The federal funds market in the financial crisis. *Journal of Finance* 66, 1109–1139.
- Bartolini, L., Hilton, S., Sundaresan, S., Tonetti, C., 2011. Collateral values by asset class: Evidence from primary securities dealers. *Review of Financial Studies* 24, 248–278.
- Bindseil, U., Nyborg, K. G., Strebulaev, I. A., 2009. Repo auctions and the market for liquidity. *Journal of Money, Credit and Banking* 41, 1391–1421.
- Bindseil, U., Weller, B., Wuertz, F., 2003. Central bank and commercial banks' liquidity management - what is the relationship? *Economic Notes* 32, 37–66.
- Brand, C., Buncic, D., Turunen, J., 2010. The impact of ECB monetary policy decisions and communication on the yield curve. *Journal of the European Economic Association* 8, 1266–1298.
- Brenner, M., Pasquariello, P., Subrahmanyam, M., 2009. On the volatility and comovement of U.S. financial markets around macroeconomic news announcements. *Journal of Financial and Quantitative Analysis* 44, 1265–1289.
- Brunnermeier, M. K., Pedersen, L. H., 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22, 2201–2238.
- Cline, W. R., 2014. Managing the Euro Area Debt Crisis. Peterson Institute for International Economics.
- Copeland, A., Martin, A., Walker, M., 2010. The Tri-Party Repo Market before the Reforms. Federal Reserve Bank of New York Staff Report 477.
- Duffie, D., 1996. Special Repo Rates. *Journal of Finance* 51, 493–526.

- European Central Bank, 2002. The liquidity management of the ECB. *Monthly Bulletin*.
- European Central Bank, 2011. The monetary policy of the ECB.
- Fecht, F., Nyborg, K. G., Rocholl, J., 2008. Liquidity management and overnight rate calendar effects: Evidence from German banks. *North American Journal of Economics and Finance* 19, 7–21.
- Fecht, F., Nyborg, K. G., Rocholl, J., 2011. The price of liquidity: The effects of market conditions and bank characteristics. *Journal of Financial Economics* 102, 344–362.
- Gorton, G., Metrick, A., 2011. Securitized Banking and the Run on Repo. *Journal of Financial Economics* 104, 425–451.
- Greene, W. H., 2008. *Econometric Analysis*. Pearson Prentice Hall, 6th ed.
- Grossman, S., 1976. On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information. *Journal of Finance* 31, 573–585.
- Hamilton, J. D., 1996. The Daily Market for Federal Funds. *Journal of Political Economy* 104, 26–56.
- Krishnamurthy, A., Nagel, S., Orlov, D., 2014. Sizing up repo. *Journal of Finance* 69, 2381–2417.
- Linzert, T., Schmidt, S., 2011. What explains the spread between the euro overnight rate and the ECB’s policy rate? *International Journal of Finance and Economics* 16, 275–289.
- Mancini, L., Ranaldo, A., Wrampelmeyer, J., 2016. The Euro Interbank Repo Market. *Review of Financial Studies* 29, 1747–1779.
- Nautz, D., Offermanns, C. J., 2007. The dynamic relationship between the euro overnight rate, the ECB’s policy rate and the term spread. *International Journal of Finance and Economics* 12, 287–300.

- Nautz, D., Offermanns, C. J., 2008. Volatility transmission in the European money market. *North American Journal of Economics and Finance* 19, 23–39.
- Nyborg, K. G., 2016. *Collateral Frameworks: The Open Secret of Central Banks*. Cambridge University Press.
- Nyborg, K. G., Bindseil, U., Strebulaev, I. A., 2002. Bidding and performance in repo auctions: Evidence from ECB open market operations. ECB Working Paper 157.
- Nyborg, K. G., Östberg, P., 2014. Money and Liquidity in Financial Markets. *Journal of Financial Economics* 112, 30–52.
- Perez-Quiros, G., Mendizabal, H. R., 2006. The daily market for funds in Europe: What has changed with the EMU? *Journal of Money, Credit and Banking* 38, 91–118.
- Rösler, C., 2017. The German electronic special repo market: Activity and prices. Working paper, University of Zurich.
- Szczerbowicz, U., 2015. The ECB unconventional monetary policies: Have they lowered market borrowing costs for banks and governments? *International Journal of Central Banking* 11, 91–127.
- Woschitz, J., 2017. Long-Term Central Bank Repos and Bank Rollover Risk. Working Paper, University of Zurich.

Appendix: Proofs

Proof of Lemma 1

It follows directly from the expressions (12) for Ω , (14) for \bar{r} , (16) for A , and (17) for \underline{r} , that if $u - \bar{y} \geq \rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\}$ then $\Omega = A$ and $\bar{r} = \underline{r}$. The expression (18) in the case that $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)$ then follows directly from (17). The expression (19) in the case that $\rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\} \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0)$ also follows from (17) after substituting in the expression for ω^* from (9). To complete the proof, we need to show that the expressions for r in (18) and (19) are less than or equal to u .

Suppose first that $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)$. Then (18) applies. Now, (18) implies that $r \leq u$ if and only if $(1 - \varepsilon_0)\hat{y}(1) + \varepsilon_0 u \leq u$, which, in turn, holds if and only if $\hat{y}(1) \leq u$ (since $\varepsilon_0 < 1$). Thus, using (4), we have that $r \leq u$ if and only if $\bar{y} + \rho\sigma_y^2(1 - \varepsilon_0)/2 \leq u$, or $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)/2$, which holds by assumption.

Suppose next that $\rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\} \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0)$. Then (19) applies, and $r \leq u$ follows immediately (because $1 - h > 0$). \square

Proof of Theorem 1

Case 1, $\eta > \kappa$: If (21) holds, that is, $u - \bar{y} \geq \rho\sigma_y^2(1 - \kappa)$, by Lemma 1, there is a unique equilibrium repo rate $r \leq u$ (as a function of parameter values). To complete the proof, we need to show that the bound (21) is also a necessary condition for equilibrium $r \leq u$.

Suppose, therefore, that $u - \bar{y} < \rho\sigma_y^2(1 - \kappa)$. By (16), we have $A = (1 - \kappa)/(1 - \varepsilon_0)$. Suppose, by contradiction to what we want to show, that there is $r \leq u$.

Consider first the case that $u - \bar{y} < \rho\sigma_y^2(1 - \eta)$ so that $\Omega = (1 - \eta)/(1 - \varepsilon_0)$, by (12). Equilibrium $r \leq u$ requires \bar{r} , as given by (14), to be at least large as \underline{r} , as given by (17). Using these equations and simplifying, this is equivalent to saying that

$$(1 - \eta)\hat{y}\left(\frac{1 - \eta}{1 - \varepsilon_0}\right) + \eta u \geq (1 - \kappa)\hat{y}\left(\frac{1 - \kappa}{1 - \varepsilon_0}\right) + \kappa u. \quad (43)$$

This can be written as $f(\eta) \geq f(\kappa)$, where $f(x) = (1 - x)\hat{y}((1 - x)/(1 - \varepsilon_0)) + xu$. From (4),

$$\hat{y}\left(\frac{1 - \eta}{1 - \varepsilon_0}\right) = \bar{y} + \frac{\rho}{2}\sigma_y^2(1 - \eta). \quad (44)$$

Thus, $f'(x) = -\bar{y} - \rho\sigma_y^2(1-x) + u$. Hence, $f'(x) < 0$ if and only if $u - \bar{y} < \rho\sigma_y^2(1-x)$. This holds by assumption on the interval $[\kappa, \eta]$. Thus, $f(\eta) < f(\kappa)$. This is equivalent to saying that (43) does not hold and contradicts that there is equilibrium $r \leq u$.

Consider next the case that $u - \bar{y} \geq \rho\sigma_y^2(1-\eta)$ so that $\Omega = \omega^*$ as given by (9). Thus,

$$\bar{r} = u - \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1-h)},$$

and the equilibrium condition $\bar{r} \geq \underline{r}$ becomes

$$u - \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1-h)} \geq \frac{(1-\kappa)(\bar{y} + \frac{1}{2}\rho\sigma_y^2(1-\kappa)) + (\kappa-h)u}{1-h}. \quad (45)$$

This can be written as

$$(u - \bar{y})^2 - 2\rho\sigma_y^2(1-\kappa)(u - \bar{y}) + \rho^2\sigma_y^4(1-\kappa)^2 \leq 0, \quad (46)$$

or,

$$((u - \bar{y}) - \rho\sigma_y^2(1-\kappa))^2 \leq 0, \quad (47)$$

which is not possible (because $\rho\sigma_y^2(1-\kappa) \neq (u - \bar{y})$). Hence, there is no equilibrium $r \leq u$ when $\kappa < \eta$ and $u - \bar{y} < \rho\sigma_y^2(1-\kappa)$. Thus, we have shown that the bound (21) is also a necessary condition for equilibrium $r \leq u$ when $\eta > \kappa$.

Case 2, $\eta \leq \kappa$: If $u - \bar{y} \geq \rho\sigma_y^2(1-\eta)$, by Lemma 1, there is a unique equilibrium repo rate $r \leq u$ (as a function of parameter values). This takes care of Case 2a in the statement of the theorem.

The remainder of the proof concerns Case 2b, that is, $u - \bar{y} < \rho\sigma_y^2(1-\eta)$. In dealing with this case, we will also establish the bound (22) as a necessary and sufficient condition for equilibrium $r \leq u$. (The bound is automatically satisfied in Case 2a). The proof proceeds by first deriving the equation for \underline{r} in the statement of the theorem, namely (25). We then establish (22) as a necessary condition for $r \leq u$ by checking the condition $\underline{r} \leq u$. Following this, we derive the equations for the upper bound, R , (24) and (23), in the statement of the theorem, in that order. Sufficiency of (22) follows thereafter by checking that $\underline{r} \leq R$.

Suppose in the remainder of this proof that $u - \bar{y} < \rho\sigma_y^2(1-\eta)$. Recall that \underline{r} is given by (17). Substituting in the values of A from (16), we get the expressions for \underline{r} in (25). It

is immediate that $\underline{r} < u$ if $u - \bar{y} \geq \rho\sigma_y^2(1 - \kappa)$. Suppose next that $u - \bar{y} < \rho\sigma_y^2(1 - \kappa)$. From the associated expression for \underline{r} in (25), we get, after some algebra, that $\underline{r} \leq u$ if and only if $\hat{y} \left(\frac{1-\kappa}{1-\varepsilon_0} \right) = \bar{y} + \frac{\rho}{2}\sigma_y^2(1 - \kappa) \leq u$. In other words, (22) is a necessary condition for $r \leq u$.

Next, consider \bar{r} . By (12), we have $\Omega = (1 - \eta)/(1 - \varepsilon_0)$. Substituting this into the expression for \bar{r} in (14), we get the first expression for \bar{r} in (24). The second expression follows from (44). Using this expression, we see that $\bar{r} < u$ if and only if $\bar{y} + \rho\sigma_y^2(1 - \eta)/2 < u$. Thus, we get (23).

Suppose now (22) holds so that $\underline{r} \leq u$. We need to show that $\underline{r} \leq R$, where $R = \min\{u, \bar{r}\}$. If so, any $r \in [\underline{r}, R]$ would be an equilibrium repo rate (of at most u), which is what we want to show. Note first that by (23), $\underline{r} \leq u$ implies $\underline{r} \leq R$ for $u - \bar{y} \leq \frac{\rho}{2}\sigma_y^2(1 - \eta)$. Suppose, therefore, that $u - \bar{y} \geq \frac{\rho}{2}\sigma_y^2(1 - \eta)$. By (25) and since $\eta \leq \kappa$, there are two cases:

Case (a): $1 - \kappa \leq (1 - \eta)/2$.

We need to show that $\underline{r} \leq \bar{r}$. Since $\rho\sigma_y^2(1 - \eta) > u - \bar{y} \geq \frac{\rho}{2}\sigma_y^2(1 - \eta)$, this is, using (23)–(25),³⁶

$$u - \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1 - h)} \leq \frac{(1 - \eta) \left(\bar{y} + \frac{1}{2}\rho\sigma_y^2(1 - \eta) \right) + (\eta - h)u}{1 - h}. \quad (48)$$

This is just the reverse of (45) in the proof of Case 1 above, with η in place of κ , and, therefore, holds.

Case (b): $1 - \kappa > (1 - \eta)/2$.

We need to show that $\underline{r} \leq \bar{r}$. There are two subcases. First, $\rho\sigma_y^2(1 - \eta) > u - \bar{y} \geq \rho\sigma_y^2(1 - \kappa)$.

In this case, $\underline{r} \leq \bar{r}$ can once again be written as (48), which holds.

Second,

$$\frac{u - \bar{y}}{\rho\sigma_y^2} \in \left[\frac{1 - \eta}{2}, 1 - \kappa \right). \quad (49)$$

Using (12), (14), (16), and (17)), the condition $\underline{r} \leq \bar{r}$ can now be written

³⁶The left hand side of (48) is \underline{r} when $u - \bar{y} \geq \rho\sigma_y^2(1 - \kappa)$ as given in (25). The right hand side is \bar{r} for $\rho\sigma_y^2(1 - \eta) > u - \bar{y} > \rho\sigma_y^2(1 - \eta)/2$ as given in (24).

$$(1 - \kappa)\hat{y} \left(\frac{1 - \kappa}{1 - \varepsilon_0} \right) + \kappa u \leq (1 - \eta)\hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right) + \eta u. \quad (50)$$

This is just (43) in the proof of Case 1 above, but with η and κ having switched places (now $\eta \leq \kappa$). Thus, (50) holds. In other words, $\underline{r} \leq \bar{r}$ given (49).

The above establishes that $\underline{r} \leq R$ if (22) holds. It follows immediately that if (22) holds, then any $r \in [\underline{r}, R]$ is an equilibrium repo rate, $r \leq u$. This completes the proof. \square

Proof of Lemma 2

Under Alternative 2, the short's optimal trade cannot lead to a higher cost of liquidity than if she chooses any arbitrary trade that gives her the requisite unit of liquidity. Consider $\omega = (1 - \eta)/(1 - \varepsilon_0)$ and borrowings of η at the unsecured rate. Thus, using the same line of argument as in the derivation of (13), we get that for the short to be willing to do a repo, we must have

$$(1 - \eta)r + \eta u \leq (1 - \eta)\hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right) + \eta u, \quad (51)$$

or

$$r \leq \hat{y} \left(\frac{1 - \eta}{1 - \varepsilon_0} \right) = \bar{y} + \frac{\rho}{2}\sigma_y^2(1 - \eta). \quad (52)$$

(28) follows from $u < r$. Statements (i) and (ii) follow immediately. \square

Proof of Lemma 3

By Assumption 6, the unsecured borrowing constraint of the cash provider is $\alpha(1 - \varepsilon_0) \geq 1 - \eta - (\kappa - \eta) = 1 - \kappa$. Since, we must also have $\phi \geq \alpha$ (feasibility), it follows that $\phi(1 - \varepsilon_0) \geq 1 - \kappa$, which is what we wanted to show. \square

Proof of Theorem 2

By Lemma 2, (28) is a necessary condition for equilibrium $r > u$. Suppose, henceforth, that this holds, that is, $u - \bar{y} < \frac{1}{2}\rho\sigma_y^2(1 - \eta)$. By (31), this is equivalent to saying that $u < \bar{r}_{neg}$. To show that equilibrium $r > u$ exists, therefore, we only need to show that $\underline{r}_{neg} \leq \bar{r}_{neg}$. If this holds as a strict inequality, any repo rate in the interval $(R_{neg}, \bar{r}_{neg}]$, where $R_{neg} = \max\{u, \underline{r}_{neg}\}$, is an equilibrium $r > u$. If $\underline{r}_{neg} = \bar{r}_{neg}$, then this common rate

is a unique equilibrium $r > u$.

Case 1, $\eta \leq \kappa$:

Claim: $\underline{r}_{neg} < \bar{r}_{neg}$ if $\eta < \kappa$ and $\underline{r}_{neg} = \bar{r}_{neg} = \hat{y} \left(\frac{1-\eta}{1-\varepsilon_0} \right)$ if $\eta = \kappa$.

To prove the claim, suppose first that $\eta < \kappa$. There are two cases to consider.

Case (a): $1 - \kappa \geq (1 - \eta)/2$. In this case, by (32), $A = (1 - \kappa)/(1 - \varepsilon_0)$. By (34), the minimum acceptable rate to the cash provider is, therefore,

$$\underline{r}_{neg} = \frac{(1 - \kappa)\hat{y} \left(\frac{1-\kappa}{1-\varepsilon_0} \right) + \kappa u - \eta u}{1 - \eta} \quad (53)$$

Thus, using (31), we have that $\underline{r}_{neg} < \bar{r}_{neg}$ if and only if (50) [from the proof of Theorem 1] holds as a strict inequality, which it does here because $\eta < \kappa$.

Case (b): $1 - \kappa < (1 - \eta)/2$.

We need only consider $u - \bar{y} > \rho\sigma_y^2(1 - \kappa)$, as the reverse is covered by the argument in Case (a). Suppose first that $u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0)\phi$ so that $A = \omega^*$. Thus, using (4), (9), and (34), we have

$$\underline{r}_{neg} = u - \frac{1}{2} \frac{(u - \bar{y})^2}{\rho\sigma_y^2(1 - \eta)}. \quad (54)$$

Hence, $\underline{r}_{neg} < u < \bar{r}_{neg}$.

Suppose next that $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)\phi$ so that $A = \phi = (1 - \eta)/(1 - h)$. Using (34), we have

$$\underline{r}_{neg} = \frac{\phi(1 - \varepsilon_0)\hat{y}(\phi) + (1 - \phi(1 - \varepsilon_0))u - \eta u}{1 - \eta}. \quad (55)$$

Hence, $\underline{r}_{neg} < u$ if and only if $\hat{y}(\phi) < u$, which is equivalent to $\bar{y} + \frac{1}{2}\rho\sigma_y^2\phi(1 - \varepsilon_0) < u$. This holds since $u - \bar{y} \geq \rho\sigma_y^2\phi(1 - \varepsilon_0)$ (by assumption). Hence, $\underline{r}_{neg} < u < \bar{r}_{neg}$. Thus, we have proved the claim above when $\eta < \kappa$.

Suppose next that $\eta = \kappa$. Now the same algebra as in Case (a) above establishes that $\underline{r}_{neg} = \bar{r}_{neg} = \hat{y} \left(\frac{1-\eta}{1-\varepsilon_0} \right)$, as claimed.

Finally, note that the expressions for \underline{r}_{neg} in the different cases above jointly establish (35).

Case 2, $\eta > \kappa$: By Lemma 3, we only need to consider the case that $(1 - \varepsilon_0)\phi \geq (1 - \kappa)$. By Lemma 2, (29) is now a second necessary condition for equilibrium $r > u$. Suppose, it

holds, that is, $u - \bar{y} < \rho\sigma_y^2(1 - \kappa)$. Thus, by (32), $A = (1 - \kappa)/(1 - \varepsilon_0)$. By (28), we also have $u - \bar{y} < \rho\sigma_y^2(1 - \eta)$ so that \bar{r}_{neg} is given by (31). The same algebra as in Case 1(a) above now shows that $\underline{r}_{neg} > \bar{r}_{neg}$, since now $\eta > \kappa$. Thus, there is no equilibrium $r > u$. \square

Remark

This remark relates to the derivation of the comparative statics of the collateral spread with respect to ε_0 that will be carried out in the proofs of Propositions 1 and 2 below. In particular, note that \bar{y} and σ_y^2 are functions of ε_0 . Using the expression for \bar{y} in terms of \bar{x} , σ_x^2 , ε_1 , and ε_0 in (1), we have $\bar{y}'(\varepsilon_0) = (1 + \bar{x} - \varepsilon_1)/(1 - \varepsilon_0)^2 = (1 + \bar{y})/(1 - \varepsilon_0)$. Let $f(\varepsilon_0) = \sigma_y^2(\varepsilon_0)(1 - \varepsilon_0) = \sigma_x^2/(1 - \varepsilon_0)$. Hence $f'(\varepsilon_0) = \sigma_x^2/(1 - \varepsilon_0)^2 = \sigma_y^2$. Furthermore, $\sigma_y^{2'}(\varepsilon_0) = 2\sigma_y^2/(1 - \varepsilon_0)$.

Proof of Proposition 1

Suppose $u - r \geq 0$. The cases for the collateral spread in Table 4 are taken directly from Theorem 1. Associated expressions for r , \underline{r} , and \bar{r} are also in Theorem 1. These have been subtracted from u to give the expressions for the collateral spread in Table 4. From these expressions, it is immediately obvious that the partial derivatives with respect to σ_y^2 and ρ are identical. Below we verify the comparative statics. Note first that, by assumption, $1 - \varepsilon_0 > 0$, $1 - h > 0$, $1 - \eta > 0$, and $1 - \kappa > 0$.

Case 1. $u - \bar{y} \geq \rho\sigma_y^2(1 - \varepsilon_0)$.

Let $B \equiv u - \bar{y} - \frac{1}{2}\rho\sigma_y^2(1 - \varepsilon_0)$. Note that in Case 1, $B > 0$. Now, using the expression for $u - r$ in Table 4, we have: $\frac{\partial(u-r)}{\partial u} = \frac{1-\varepsilon_0}{1-h} > 0$; $\frac{\partial(u-r)}{\partial \bar{y}} = -\frac{1-\varepsilon_0}{1-h} < 0$; $\frac{\partial(u-r)}{\partial h} = \frac{1-\varepsilon_0}{(1-h)^2}B > 0$; $\frac{\partial(u-r)}{\partial \sigma_y^2} = -\frac{1}{2}\frac{(1-\varepsilon_0)^2}{1-h} < 0$; and, using the expressions in the remark above,

$$\frac{\partial(u-r)}{\partial \varepsilon_0} = \frac{-B}{1-h} + \frac{1-\varepsilon_0}{1-h} [-\bar{y}'(\varepsilon_0) - f'(\varepsilon_0)] = \frac{-u-1}{1-h} < 0$$

since $u > -1$. This concludes Case 1.

Case 2: $\rho\sigma_y^2 \max\{1 - \eta, 1 - \kappa\} \leq u - \bar{y} < \rho\sigma_y^2(1 - \varepsilon_0)$.

Note that in Case 2, $u - \bar{y} > 0$. Using the expression for $u - r$ in Table 4, we have: $\frac{\partial(u-r)}{\partial u} = \frac{u-\bar{y}}{\rho\sigma_y^2(1-h)} > 0$; $\frac{\partial(u-r)}{\partial \bar{y}} = -\frac{u-\bar{y}}{\rho\sigma_y^2(1-h)} < 0$; $\frac{\partial(u-r)}{\partial h} = \frac{1}{2}\frac{(u-\bar{y})^2}{\rho\sigma_y^2(1-h)^2} > 0$; $\frac{\partial(u-r)}{\partial \sigma_y^2} = -\frac{(u-\bar{y})^2}{\rho\sigma_y^4(1-h)} < 0$;

and, using the expressions in the remark above

$$\frac{\partial(u-r)}{\partial\varepsilon_0} = \frac{1}{\sigma_y^2\rho(1-h)} \left[-(u-\bar{y})\bar{y}'(\varepsilon_0) - \frac{(u-\bar{y})^2}{1-\varepsilon_0} \right] = \frac{u-\bar{y}}{\sigma_y^2(1-\varepsilon_0)\rho(1-h)} [-u-1] < 0.$$

This concludes Case 2.

Case 3a: $\eta \leq \kappa$ and $\frac{1}{2}\rho\sigma_y^2(1-\eta) \leq u-\bar{y} < \rho\sigma_y^2(1-\eta)$.

Using the expression for $u-\bar{r}$ in Table 4, we have $\frac{\partial(u-\bar{r})}{\partial u} = \frac{1-\eta}{1-h} > 0$; $\frac{\partial(u-\bar{r})}{\partial\bar{y}} = -\frac{1-\eta}{1-h} < 0$; $\frac{\partial(u-\bar{r})}{\partial h} = \frac{1-\eta}{(1-h)^2} [u-\bar{y} - \frac{1}{2}\rho\sigma_y^2(1-\eta)] > 0$; $\frac{\partial(u-\bar{r})}{\partial\sigma_y^2} = -\frac{1}{2}\rho\frac{(1-\eta)^2}{1-h} < 0$; and, using the expressions in the remark above

$$\frac{\partial(u-\bar{r})}{\partial\varepsilon_0} = \frac{1-\eta}{1-h} \left[-\frac{1+\bar{y}}{1-\varepsilon_0} - \frac{\rho\sigma_y^2(1-\eta)}{1-\varepsilon_0} \right] < 0,$$

since $\bar{y} > -1$. This concludes Case 3a.

Observe now that the expression, and therefore also the comparative statics, for $u-\underline{r}$ in Case 3b is identical to that for $u-r$ in Case 2. Finally, the expression for $u-\underline{r}$ in Case 3c is the same as for $u-\bar{r}$ in Case 3a, but with κ in place of η . The comparative statics are, therefore, analogous. \square

Proof of Proposition 2

Suppose $u-r < 0$. The cases for the collateral spread in Table 5 are taken directly from Theorem 2. Associated expressions for r , \underline{r}_{neg} , and \bar{r}_{neg} are also in Theorem 2. These have been subtracted from u to give the expressions for the collateral spread in Table 5. From these expressions, it is immediately obvious that the partial derivatives with respect to σ_y^2 and ρ are identical. Below we verify the comparative statics. Note first that, by assumption, $1-\varepsilon_0 > 0$, $1-h > 0$, $1-\eta > 0$, and $1-\kappa > 0$.

Case 1a: $\eta \leq \kappa$.

Using the expression for $u-\bar{r}_{neg}$ in Table 5, we have: $\frac{\partial(u-\bar{r}_{neg})}{\partial u} = 1$; $\frac{\partial(u-\bar{r}_{neg})}{\partial\bar{y}} = -1$; $\frac{\partial(u-\bar{r}_{neg})}{\partial h} = 0$; $\frac{\partial(u-\bar{r}_{neg})}{\partial\sigma_y^2} = -\frac{1}{2}\rho(1-\eta) < 0$; and using the expressions in the remark above $\frac{\partial(u-\bar{r}_{neg})}{\partial\varepsilon_0} = -\sigma_y^2(1-\eta)/(1-\varepsilon_0) < 0$. This concludes Case 1a.

Case 1b: $\eta \leq \kappa$ and $u-\bar{y} \geq \rho\sigma_y^2(1-\varepsilon_0)\phi$.

Let $B \equiv u-\bar{y} - \frac{1}{2}\rho\sigma_y^2(1-\varepsilon_0)\phi$. Note that $B > 0$ in Case 1b. Using the expression for

$u - r_{neg}$ in Table 5, we have: $\frac{\partial(u-r_{neg})}{\partial u} = \frac{1-\varepsilon_0}{1-h} > 0$ and $\frac{\partial(u-r_{neg})}{\partial \bar{y}} = -\frac{1-\varepsilon_0}{1-h} < 0$. Recalling that $\phi = (1-\eta)/(1-h)$, we also have

$$\frac{\partial(u-r_{neg})}{\partial h} = \frac{1-\varepsilon_0}{(1-h)^2} B + \frac{1-\varepsilon_0}{1-h} \left[-\frac{1}{2} \rho \sigma_y^2 \frac{(1-\varepsilon_0)(1-\eta)}{(1-h)^2} \right] = \frac{1-\varepsilon_0}{(1-h)^2} [u - \bar{y} - \rho \sigma_y^2 (1-\varepsilon_0) \phi] \geq 0.$$

Now, $\frac{\partial(u-r_{neg})}{\partial \sigma_y^2} = -\frac{\rho}{2} \frac{(1-\varepsilon_0)^2}{(1-h)^2} \phi < 0$. Finally, using the expressions in the remark above, we have

$$\frac{\partial(u-r_{neg})}{\partial \varepsilon_0} = -\frac{B}{1-h} + \frac{1-\varepsilon_0}{1-h} \left[-\frac{1+\bar{y}}{1-\varepsilon_0} - \frac{\rho}{2} \sigma_y^2 \phi \right] < 0.$$

This concludes Case 1b.

Case 1c: $\eta \leq \kappa$, $\rho \sigma_y^2 (1-\kappa) \leq u - \bar{y} \leq \rho \sigma_y^2 (1-\varepsilon) \phi$.

This is the same as Case 2 in Proposition 1, but with $(1-\eta)$ in the denominator instead of $1-h$. All comparative statics are, therefore, the same, except for with respect to h , where the partial derivative is now equal to zero.

Case 1d: $\eta \leq \kappa$, $u - \bar{y} \leq \rho \sigma_y^2 (1-\kappa)$.

This is the same as Case 2 in Proposition 1, but with $(1-\eta)$ in the denominator instead of $1-h$. All comparative statics are, therefore, the same, except for with respect to h , where the partial derivative is now equal to zero.

Case 2: $\eta = \kappa$

The collateral spread has the same expression as in Case 1a above, and so the comparative statics are identical.

Case 3: $\eta > \kappa$ and $(1-\varepsilon_0)\phi \leq (1-\kappa)$.

The collateral spread has the same expression as in Case 1b above, and so the comparative statics are identical. \square

Appendix: Figures

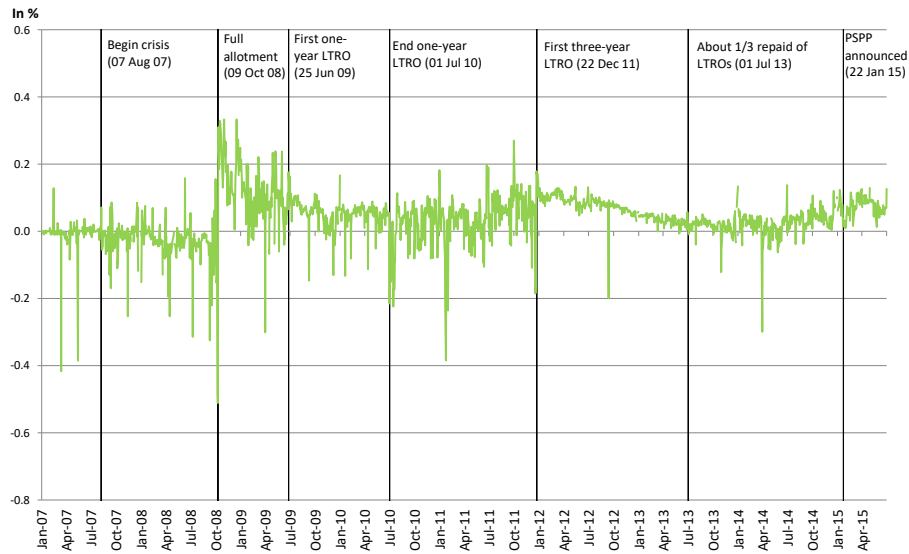


Figure 1a: Overnight collateral spread, Eonia–GCP ECB basket (in percent), January 2, 2007 to June 30, 2015. Eonia is a volume-weighted average of overnight unsecured euro transactions by reporting panel banks. See <http://www.euribor-rates.eu/eonia.asp>. The repo rate is the daily volume-weighted average of overnight transactions in Eurex Repo’s GC Pooling ECB basket. Vertical bars relate to the financial crisis and ECB unconventional monetary policies (see Section 5).

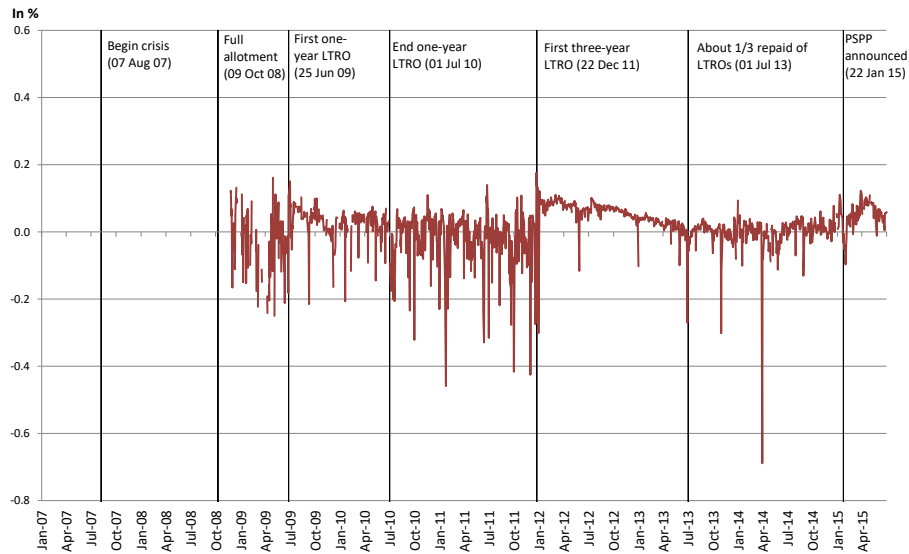


Figure 1b: Overnight collateral spread, Eonia–GCP ECB Extended basket (in percent), November 25, 2007 to June 30, 2015. The repo rate is the daily volume-weighted average of overnight transactions in Eurex Repo’s GC Pooling ECB Extended basket.

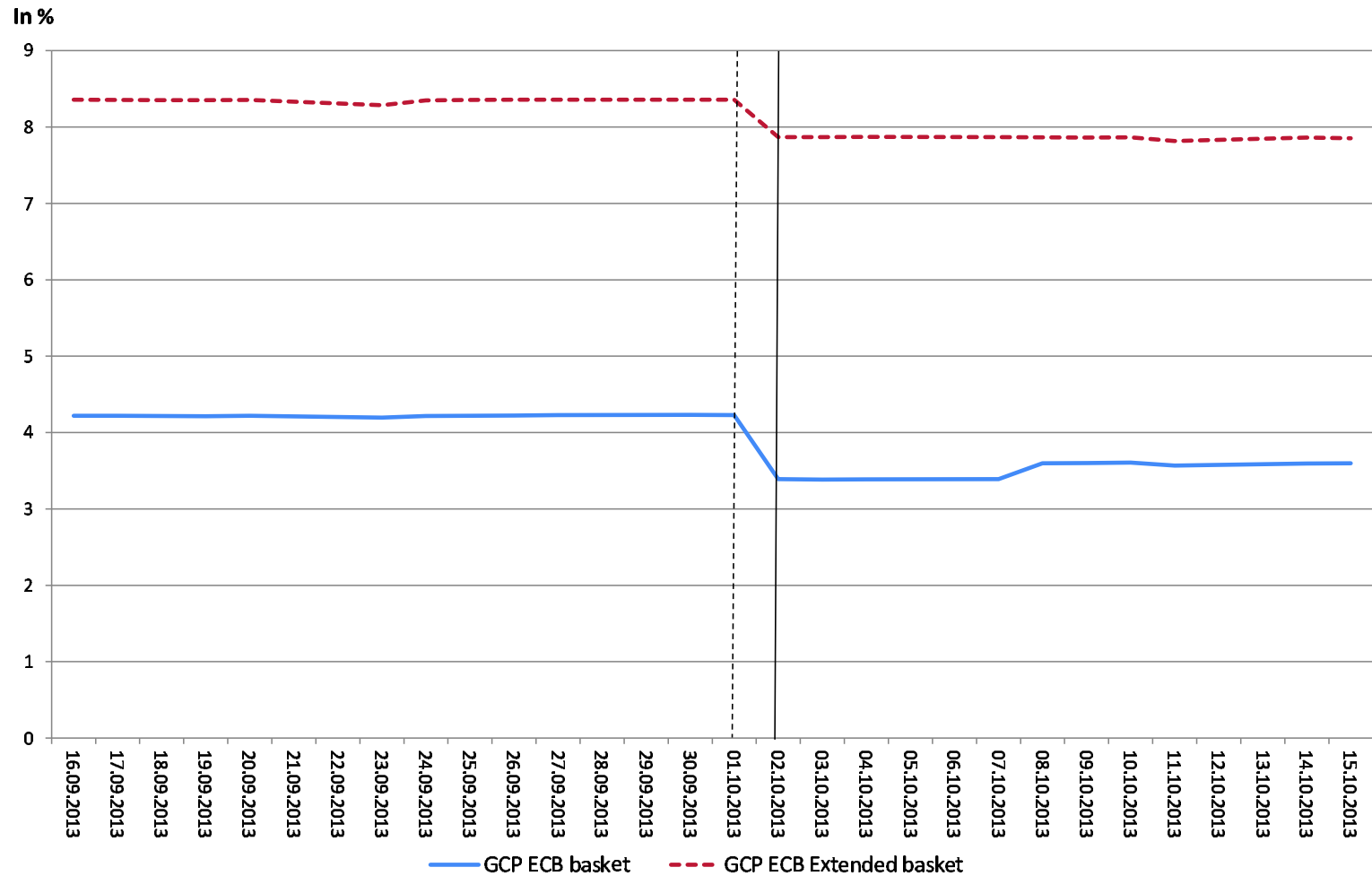


Figure 2: Daily average haircuts (in percent) applied by Eurex Repo in the two baskets, GC Pooling ECB and GC Pooling ECB Extended, September 16, 2013 to October 15, 2013. Eurosystem haircuts changed on October 1, 2013, as indicated by the dashed vertical line. Eurex Repo changed haircuts for securities in the two baskets on October 2, 2013, as indicated by the solid vertical line.

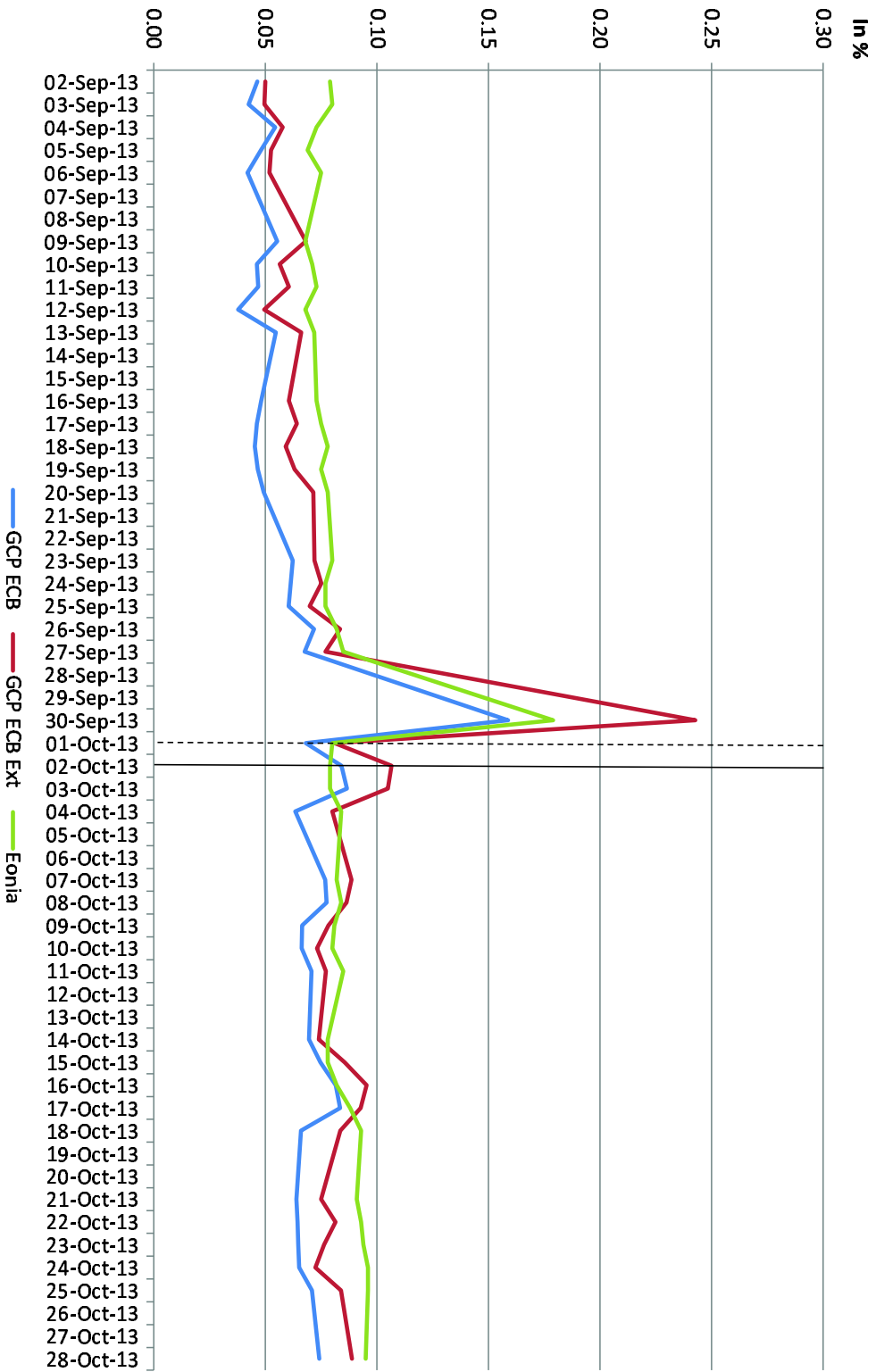


Figure 3: Eonia and GC Pooling overnight rates (daily volume-weighted averages, in percent), September 2, 2013 to October 28, 2013. Eurosystem haircuts changed on October 1, 2013, as indicated by the dashed vertical line. Eurex Repo changed haircuts for securities in the two baskets on October 2, 2013, as indicated by the solid vertical line.

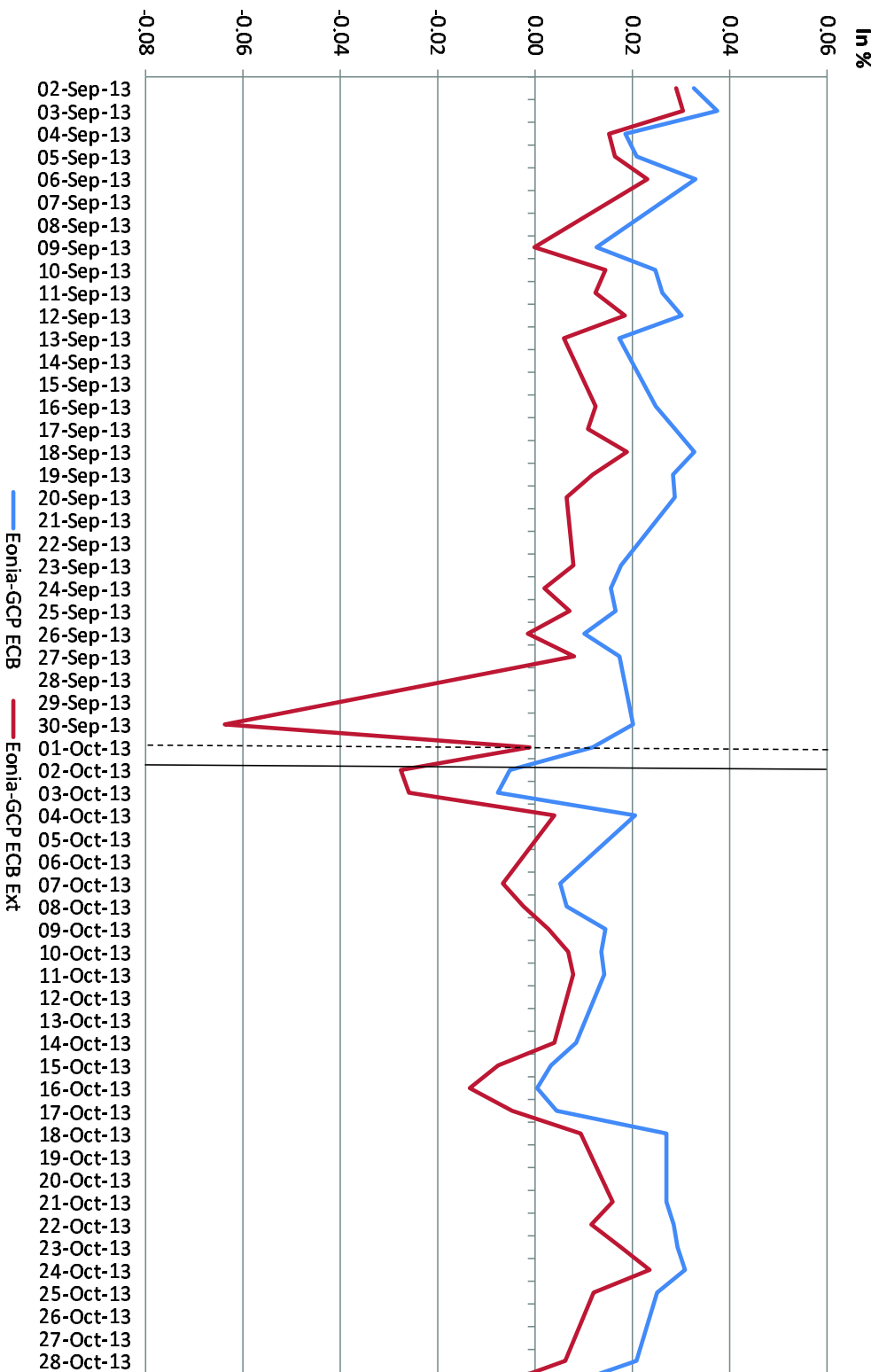


Figure 4: Overnight collateral spreads, Eonia less GC Pooling ECB and GC Pooling ECB Extended baskets, (daily volume-weighted averages, in percent), September 2, 2013 to October 28, 2013. Eurosystem haircuts changed on October 1, 2013, as indicated by the dashed vertical line. Eurex Repo changed haircuts for securities in the two baskets on October 2, 2013, as indicated by the solid vertical line.

Appendix: Tables

This appendix contains all tables for the empirical section of the paper.

Table 7: Statistics on Eurex Repo baskets

This table displays descriptive statistics on the different GC baskets of Eurex Repo for the period January 2, 2005 to June 30, 2015. The sample contains 4 GC Pooling baskets, and 24 Euro Repo baskets. The first column shows when the first and last trades in each basket occur. The second column, No. Obs., counts the number of trades. The third column, ON obs., displays the number of trades with the term overnight. The fourth column, % of total, shows the percentage of the ON transactions of all transactions. The fifth column, TN obs., displays the number of trades with the term tomorrow-next. The sixth column, % of total, is the percentage of the TN transactions of all transactions. The seventh column, Av. Rate, shows the volume-weighted average ON (TN) rate across all observations for the GC Pooling ECB and ECB Ext. (all other) baskets. Using the same tenors, the next column, Av. Coll. Spread, displays the volume-weighted collateral spread, Eonia–repo rate. For the GCP ECB and ECB Ext. (all other) baskets, we use the Eonia of the day of (after) the transaction. The ninth column, Volume - Total, is the sum of the total transacted volume across all tenors in that basket. The next column, Volume - By Trading Day, shows the average volume by trading day (period that the basket is traded). The last column, Volume - By Transact., captures the average transaction volume of each basket.

	first - last trade	No. Obs.	ON obs.	% of total	TN obs.	% of total	Av. Rate (in bps)	Av. Coll. Spr. (in bps)	Volume (in EUR million)		
									Total	By Trading day	By Transaction
GC Pooling ECB	Jan 2, 07 – Jun 30, 15	151,330	83,804	55.38	26,707	17.65	84.05	4.07	87,434,094	40,311	578
GC Pooling ECB Ext.	Nov 24, 08 – Jun 30, 15	67,944	32,846	48.34	15,181	22.34	26.12	1.38	30,908,936	18,333	455
German KfW / Laender	Jan 3, 07 – Sep 14, 12	8,457	0	0.00	3,804	44.98	100.54	-0.42	2,403,894	1,645	284
EIB / KfW	May 25, 11 – Sep 14, 12	2,399	0	0.00	1,701	70.90	42.24	11.43	541,672	1,598	226
KfW	Sep 17, 12 – Jun 30, 15	3,217	154	4.79	2,497	77.62	3.91	2.36	883,928	1,250	275
Euro Covered Bond	Jan 9, 07 – Aug 14, 14	7,474	8	0.11	4,882	65.32	151.19	-6.14	2,415,711	1,243	323
Germany	Jan 9, 07 – Jun 30, 15	3,534	107	3.03	2,054	58.12	67.85	5.28	2,444,870	1,130	692
EIB	Sep 17, 12 – Jun 29, 15	2,869	186	6.48	2,292	79.89	4.45	2.49	783,557	1,110	273
EFSF	Sep 17, 12 – Jun 30, 15	2,129	47	2.21	1,873	87.98	2.91	1.87	422,073	598	198
French Covered Bond	Apr 4, 09 – May 19, 15	4,467	35	0.78	2,792	62.50	40.20	0.84	777,149	496	174
German Jumbo Pfandbrief	Jan 2, 07 – Jul 2, 13	2,045	0	0.00	1,141	55.79	205.38	-1.58	741,682	446	363
German Laender	Sep 18, 12 – Jun 30, 15	865	48	5.55	327	37.80	2.24	3.88	279,568	396	323
Agency	Feb 9, 07 – Dec 17, 14	3,645	4	0.11	1,972	54.10	61.06	-3.35	697,299	347	191
Germany 10 Year	Jan 8, 07 – Jan 6, 15	236	0	0.00	72	30.51	258.23	15.22	679,004	332	2,877
German Corporate	Feb 18, 08 – Dec 17, 12	216	0	0.00	73	33.80	111.47	0.32	102,330	82	474
German Pfandbrief	Mar 6, 08 – Jan 15, 15	206	0	0.00	94	45.63	284.56	-5.58	103,299	59	501
GC Pooling Equity	Mar 16, 11 – Jun 23, 15	153	6	3.92	23	15.03	11.26	-19.44	38,810	36	254
Austrian Government	Dec 14, 12 – Jun 17, 15	77	9	11.69	32	41.56	-4.27	3.93	20,964	33	272
German Gov. Guaranteed	Oct 16, 12 – Sep 4, 14	40	0	0.00	38	95.00	6.05	4.08	8,841	18	221
French Government	Nov 12, 12 – Jun 16, 15	35	4	11.43	6	17.14	3.08	3.62	11,339	17	324
Finnish Government	Apr 22, 13 – Mar 18, 15	18	2	11.11	13	72.22	4.46	5.18	7,884	16	438
GC Pooling INT MXQ	Mar 05, 14 – Sep 26, 14	8	2	25.00	4	50.00	5.25	-3.57	1,840	13	230
Dutch Government	Jan 17, 14 – Jun 26, 15	18	1	5.56	7	38.89	-0.14	3.98	3,920	11	218
European Government	Jan 9, 07 – May 8, 15	150	0	0.00	17	11.33	276.09	-5.55	22,775	11	152
Belgian Government	Sep 27, 12 – Jun 10, 15	39	1	2.56	17	43.59	-0.35	2.50	6,806	10	175
European Corporate	Feb 12, 09 – Feb 19, 14	75	0	0.00	45	60.00	78.51	-8.00	9,674	8	129
Euro Gov. Guaranteed	Nov 14, 12 – Dec 17, 14	11	0	0.00	0	0.00	0.00	0.00	1,426	3	130
Spanish Government	Jun 4, 13 – Apr 14, 15	6	0	0.00	1	16.67	10.00	-1.90	606	1	101

Table 8: Descriptive statistics on the collateral spread

This table displays descriptive statistics of our sample of daily average overnight collateral spreads. For each day, the collateral spread (in bps) is calculated as the difference between the unsecured volume-weighted average overnight rate, the Eonia, and the daily volume-weighted average overnight repo rate, GCP ECB or GCP ECB Ext. rate. The sample period for Eonia – GCP ECB is January 2, 2007 to June 30, 2015, and for Eonia – GCP ECB Ext. it is November 24, 2008 to June 30, 2015. Columns 2-7 capture the number of observations (days), the mean, the standard error, the median, the standard deviation, the minimum, and the maximum. The last two columns show the negative occurrence of the collateral spread: the second last column counts the number of negative days observed, and the last column displays the percentage of negative days of the total number of days, when trades occur in that basket.

	No. Obs.	Mean	St. Error	Median	St. Deviation	Min	Max	# negative days	% of neg. days of obs.
Eonia – GCP ECB rate	2,167	3.773	0.145	4.109	6.743	-51.135	33.271	496	22.889%
Eonia – GCP ECB Ext. rate	1,604	1.362	0.167	2.371	6.685	-68.919	17.618	461	28.741%

Table 9: Standardized Eonia at end of maintenance period and month

This table displays statistics on the standardized Eonia for the five last days of the maintenance period and the last day of the month. `endmp` to `endmp-4` denote the last five days of the maintenance period, where `endmp` is the last day and `endmp-4` is the fourth day before the end. `monthend` refers to the last trading day of the month. The sample period is January 2, 2007 to August 29, 2008, representing twenty-one maintenance periods. However, the last maintenance period does not include `endmp-4` to `endmp`, and the first one does not capture `monthend`, yielding 20 observations for each of these days. Eonia is the volume-weighted average unsecured overnight rate. For each maintenance period, we compute the standardized Eonia, `stand. Eonia`, as defined by (38), excluding data from the five last days of the maintenance period and the last day of all calendar months. The third column shows the average of the standardized Eonia across maintenance periods on the indicated days. The fourth column shows the averages in terms of absolute values. The fifth (sixth) column, positive (negative), displays the average value, when the standardized Eonia is positive (negative). The column `upspike-total` (`downspike-total`) shows the number of times when the standardized Eonia is positive (negative). The column `upspike-abs. >2` (`downspike-abs. >2`) displays the number of times when the absolute standardized Eonia exceeds two.

	No. Obs.	average value	stand. Eonia			upspike		downspike	
			absolute value	positive	negative	total	abs. >2	total	abs. >2
<code>endmp</code>	20	-7.675	16.310	9.594	-21.805	9	5	11	7
<code>endmp-1</code>	20	-3.585	5.012	4.759	-5.057	3	1	17	9
<code>endmp-2</code>	20	-4.181	4.650	1.566	-5.195	3	1	17	8
<code>endmp-3</code>	20	-3.267	3.766	1.247	-4.395	4	1	16	7
<code>endmp-4</code>	20	-1.474	2.415	1.568	-2.778	6	2	14	8
<code>monthend</code>	20	3.583	3.585	3.773	-0.021	19	13	1	0

Table 10: Standardized Eonia rankings, end of maintenance period and month

This table shows the rankings of the standardized Eonia on the five last days of each maintenance period, endmp to endmp-4, and the last day of each calendar month, monthend. The sample period is January 2, 2007 to August 12, 2008, representing twenty maintenance periods (the first maintenance period starts on December 13, 2006). Eonia is the volume-weighted average unsecured overnight rate. For each maintenance period, we compute the standardized Eonia, stand. Eonia, as defined by (38), excluding data from the five last days of the maintenance period and the last day of all calendar months. For the last five days of the maintenance period, the ranking order is determined by the sign of the collateral spread on the last day of the maintenance period, endmp. If it is negative (positive), all values in the maintenance period are ranked in ascending (descending) order, starting with the lowest negative (highest) value. For the last day of the month, the ranking order is always descending from the highest value. Days in the last week of the maintenance period that are ranked first are marked in bold.

	Period 1		Period 2		Period 3		Period 4		Period 5		Period 6		Period 7	
	stand. Eonia	rank	stand. Eonia	rank	stand. Eonia	rank	stand. Eonia	rank	stand. Eonia	rank	stand. Eonia	rank	stand. Eonia	rank
endmp	-5.51	1	23.37	1	-107.52	1	-2.45	3	-113.47	1	-0.85	5	0.13	13
endmp-1	-4.27	3	-2.80	17	-11.24	2	-9.77	1	-38.96	3	-0.20	7	-0.78	8
endmp-2	-5.51	2	-4.44	18	-11.24	3	-6.84	2	-41.53	2	0.13	9	-0.78	7
endmp-3	-4.27	4	-6.08	19	-6.54	4	-0.26	12	-33.83	4	-4.14	2	-1.69	3
endmp-4	-3.64	5	-6.08	20	-4.19	5	0.47	19	-5.57	5	-4.47	1	-5.33	1
monthend	-	-	3.74	2	7.55	1	5.60	1	9.85	1	1.12	1	6.50	1
	Period 8		Period 9		Period 10		Period 11		Period 12		Period 13		Period 14	
endmp	28.01	1	-1.22	5	-0.09	10	-3.61	1	0.05	9	1.20	5	1.44	4
endmp-1	0.37	5	-3.29	1	-1.52	3	-2.44	4	-3.03	4	-0.29	15	-0.67	16
endmp-2	-3.08	17	-1.53	3	-1.29	5	-0.92	5	-4.19	2	-0.27	13	-0.65	15
endmp-3	-4.81	20	-0.05	16	-0.68	6	0.38	15	-5.11	1	1.02	6	-0.29	11
endmp-4	-3.08	18	3.36	25	-0.53	8	0.46	21	-4.12	3	1.93	1	-0.16	9
monthend	2.10	2	1.48	3	0.87	3	1.74	1	0.05	10	-0.02	10	3.14	1
	Period 15		Period 16		Period 17		Period 18		Period 19		Period 20			
endmp	24.85	1	-3.01	1	-0.06	9	4.12	1	-2.04	2	3.18	2		
endmp-1	13.20	2	-0.74	6	-3.81	1	-1.49	19	-0.67	4	0.70	12		
endmp-2	3.86	3	-0.91	5	-2.95	2	-1.82	20	-0.36	8	0.70	11		
endmp-3	0.63	7	-0.45	8	-0.40	7	-1.35	18	-0.39	7	2.96	3		
endmp-4	-0.06	11	-0.35	9	0.54	12	-0.77	12	-0.54	6	2.64	4		
monthend	3.17	4	1.16	4	2.75	1	2.33	2	3.32	1	9.96	1		

Table 11: Spikes test - regressions

This table shows time-series regressions of the overnight collateral spread, Eonia-GCP ECB (in bps), on indicator variables of spikes in the Eonia. The unsecured rate, Eonia, and the secured rate, GCP ECB, are daily volume-weighted overnight averages. The sample period is January 2, 2007 to August 29, 2008. The independent variables are as follows: perc10 (perc90) is a dummy variable that takes the value of one when the Eonia is in the lower 10% (upper 90%) in the respective maintenance period, excluding the last day of the month. The indicator variable perc10|nonendres (perc90|nonendres) is one if perc10 (perc90) is one and the day is not one of the last five days of the maintenance period. The indicator variable perc10|endres (perc90|endres) is one if perc10 (perc90) is one and the day is one of the last five days of the maintenance period. fincrisis is a dummy variable that is equal to one from August 7, 2007 to August 29, 2008. monthend is an indicator variable for the last trading day of the month. The following regression is run: $y_t = \beta_0 + \beta_1 \text{perc10|nonendres}_t + \beta_2 \text{perc90|nonendres}_t + \beta_3 \text{perc10|endres}_t + \beta_4 \text{perc90|endres}_t + \beta_5 \text{fincrisis}_t + \beta_6 \text{monthend}_t + \varepsilon_t$. Newey-West standard errors with five lags (Greene, 2008) are in parentheses. The superscripts *, **, and *** indicate significance at the 10%, 5% and 1% levels.

	Eonia – GCP ECB	Eonia – GCP ECB
constant	-0.241 (0.292)	-0.062 (0.257)
perc10	-3.191** (1.394)	
perc90	2.349* (1.307)	
perc10 nonendres		-0.556 (1.337)
perc90 nonendres		-0.056 (0.750)
perc10 endres		-5.212** (2.191)
perc90 endres		6.143*** (2.367)
fincrisis	-2.186*** (0.544)	-2.469*** (0.521)
monthend	0.960 (0.833)	0.964 (0.836)
No. Obs.	422	422
Adj. R ²	0.914	0.132

Table 12: Changes in Eurosystem Haircuts

This table shows the changes in Eurosystem haircuts (in percentage points) on October 1, 2013 for marketable securities. This is based on securities' liquidity categories, maturity buckets, and ratings. The changes in haircuts are computed from Tables 5.3 and Tables 5.4 in Nyborg (2016). There are five liquidity categories, briefly summarized as: I) government securities, II) local and regional government securities as well Jumbo-style supranational/agency bonds, III) corporate, non-Jumbo and financial securities, IV) unsecured bank bonds, V) asset-backed securities. "fixed" and "zero" indicate fixed and zero coupon instruments, respectively. Eligible floating rate instruments in Categories I to IV receive the same haircuts as what is applied to an instrument with in the same category and the same rating in the zero to one year maturity bucket.

Bonds rated AAA to A-, Haircut changes	Cat I		Cat II		Cat III		Cat IV		Cat V
Time to maturity	fixed	zero	fixed	zero	fixed	zero	fixed	zero	
0-1	0.0	0.0	0.0	0.0	-0.5	-0.5	0.0	0.0	-6.0
1-3	-0.5	0.5	-1.0	0.0	-1.0	0.0	0.0	0.0	-6.0
3-5	-1.0	-0.5	-1.0	-0.5	-2.0	-1.0	0.0	0.0	-6.0
5-7	-1.0	-0.5	-1.0	-0.5	-2.0	-1.5	0.0	0.0	-6.0
7-10	-1.0	-0.5	-1.0	0.0	-2.5	-1.5	0.0	0.0	-6.0
>10	-0.5	-1.5	0.5	-1.5	-2.0	-3.5	0.0	0.0	-6.0
Bonds rated BBB+ to BBB-									
Time to maturity	fixed	zero	fixed	zero	fixed	zero	fixed	zero	fixed
0-1	0.5	0.5	1.0	1.0	0.0	0.0	-2.0	-2.0	-10.0
1-3	0.5	1.5	-0.5	3.0	-3.0	-3.0	-3.0	-3.0	-10.0
3-5	1.5	2.0	0.0	3.5	-3.0	-3.0	-4.0	-3.0	-10.0
5-7	2.0	3.0	-2.0	1.5	-2.0	-1.5	-2.5	-3.0	-10.0
7-10	2.5	3.5	-1.0	5.0	-2.0	-1.0	-2.0	-2.0	-10.0
>10	2.5	2.5	2.5	4.0	-2.0	-3.0	-2.0	-2.0	-10.0

Table 13: GCP ECB basket - distribution of securities

This table displays the distribution of the securities contained in the GC Pooling ECB basket across the ECB liquidity categories and maturity buckets on September 26, 2013. There are five liquidity categories, briefly summarized as I) government securities, II) local and regional government securities as well Jumbo-style supranational/agency bonds, III) corporate, non-Jumbo and financial securities, IV) unsecured bank bonds, V) asset-backed securities.

A- to AAA	Liquidity Group				
Years to Maturity	Cat I	Cat II	Cat III	Cat IV	Total
0-1	177	1,103	1,469	132	2,881
1-3	137	444	711	38	1,330
3-5	120	335	522	33	1,010
5-7	99	217	310	20	646
7-10	121	207	274	13	615
> 10	369	170	130	14	683
Total	1,023	2,476	3,416	250	7,165

Table 14: GCP ECB Extended basket - distribution of securities

This table displays the distribution of the securities contained in the GC Pooling ECB Extended basket across the Eurosystem liquidity categories, maturity buckets and ratings on September 26, 2013. There are five liquidity categories, briefly summarized as: I) government securities, II) local and regional government securities as well Jumbo-style supranational/agency bonds, III) corporate, non-Jumbo and financial securities, IV) unsecured bank bonds, V) asset-backed securities.

A- to AAA	Liquidity Group				
Years to Maturity	Cat I	Cat II	Cat III	Cat IV	Total
0-1	291	1,227	1,705	4,682	7,905
1-3	186	487	863	2,996	4,532
3-5	160	373	634	1,911	3,078
5-7	133	244	396	938	1,711
7-10	156	222	376	608	1,362
> 10	446	184	270	252	1,152
Total	1,372	2,737	4,244	11,387	19,740
BBB- to BBB+	Liquidity Group				
Years to Maturity	Cat I	Cat II	Cat III	Cat IV	Total
0-1	12	2	99	265	378
1-3	9	3	69	187	268
3-5	7		68	117	192
5-7	8	1	71	64	144
7-10	10		41	50	101
> 10	39		55	56	150
Total	85	6	403	739	1,233

Table 15: Effect of haircut changes on the collateral spread

This table shows output from the regression: $y_t = \beta_0 + \beta_1 \text{newhaircut}_t + \beta_2 \text{endmp1}_1 + \beta_3 \text{endmp2}_t + \varepsilon_t$, where y is the Eonia-GCP ECB or the Eonia-GCP ECB Ext. (both in bps). The ECB announced haircut changes on September 27, 2013 starting October 1, 2013. Haircuts in the GCP ECB and ECB Ext. baskets were changed by Eurex Repo on October 2, 2013. We run the regression over periods of two weeks (Sep 13 – Oct 14, 2013), three weeks (Sep 6 – Oct 21, 2013), and four weeks (Aug 29 – Oct 28, 2013) around the haircut change. The unsecured rate, Eonia, and the secured rates, GCP ECB, and GCP ECB Ext., are daily volume-weighted overnight averages. The dummy variable newhaircut is equal to one starting October 2, 2013. endmp1 and endmp2 are indicator variables for the end of the maintenance periods on September 10, 2013 and October 8, 2013, respectively. The following days are removed: the announcement day, September 27, 2013, the last trading days in August, September, and October, and the first trading day in October. Newey-West standard errors (two lags) are in parentheses. The superscripts *, **, and *** indicate significance at the 10%, 5% and 1% levels.

Sep 13, 2013 – Oct 14, 2013		
	Eonia – GCP ECB Ext	Eonia – GCP ECB
constant	0.819*** (0.236)	2.205*** (0.340)
newhaircut	-1.289** (0.572)	-1.461*** (0.412)
endmp2	0.234 (0.574)	-0.098 (0.319)
No. Obs.	20	20
Adj. R ²	0.231	0.407
Sep 6, 2013 – Oct 21, 2013		
	Eonia – GCP ECB Ext.	Eonia – GCP ECB
constant	0.969*** (0.198)	2.303*** (0.249)
newhaircut	-1.138** (0.471)	-1.201** (0.437)
endmp1	0.478** (0.198)	0.170 (0.249)
endmp2	-0.068 (0.468)	-0.456 (0.399)
No. Obs.	30	30
Adj. R ²	0.175	0.234
Aug 29, 2013 – Oct 28, 2013		
	Eonia – GCP ECB Ext.	Eonia – GCP ECB
constant	1.250*** (0.255)	2.414*** (0.208)
newhaircut	-1.097** (0.484)	-1.005** (0.425)
endmp1	0.198 (0.255)	0.059 (0.208)
endmp2	-0.389 (0.444)	-0.762* (0.398)
No. Obs.	40	40
Adj. R ²	0.145	0.171

Table 16: Volatility

This table shows output from four regressions based on the following general specification: $\Delta y_t = \beta_0 + \beta_1 \text{govcouncil_mp}_t + \beta_2 \text{govcouncil_nonmp}_t + \beta_3 \text{vstoxx}_{t-1} + \beta_4 \text{excessliq}_{t-1} + \Gamma_1' \mathbf{X}_t + \Gamma_2' \mathbf{Z}_t + \varepsilon_t$, where Δy is the change in the collateral spread Eonia-GCP ECB ($\Delta \text{Collspread}_1$) or Eonia-GCP ECB Ext. ($\Delta \text{Collspread}_2$). The unsecured rate, Eonia, and the secured rates, GCP ECB, and GCP ECB Ext., are daily volume-weighted overnight averages (in bps). All four models include, as regressors, the dummy variables `govcouncil_mp` and `govcouncil_nonmp`, which are equal to one on Governing Council meeting days where policy rates are, respectively, are not, subject to change; `vstoxx` (in index points), which is an implied volatility of options on the Euro Stoxx 50; `excessliq` (in EUR billion), which is excess Eurosystem liquidity injections as reported by the ECB; and \mathbf{X} , which is a vector of control variables for which the output is suppressed, namely, `perc10|endres`, `perc90|endres`, `monthend`, `fin crisis` (equal to one starting on August 7, 2007), and dummy variables for the first and last trading days in a month. Model 2, 3, and 4 also include a vector of indicator variables, \mathbf{Z} , to control for: settlement days of the first one-year LTRO (`1yearltro`) on June 25, 2009 and both three-year LTROs (`3yearltros`) on December 22, 2011 and March 1, 2012; the introduction of a zero deposit facility rate (`zerorate`) on July 11, 2012; the introduction and implementation of full allotment on October 9, 2008, October 13, 2008, and October 15, 2008 (`fullallot`); and the announcement days of other key unconventional monetary policies by the ECB. These announcement days, which are not reported on, involve news on EFSF/ESM, i.e., May 10, 2010, March 14, 2011, and March 26, 2011; the announcements of covered bond purchase programmes on May 7, 2009, October 6, 2011, and September 4, 2014; the implementation of very long-term LTROs, one-year and three-year, made public on May 7, 2009, and December 8, 2012; and the announcement of the public sector purchase programme on January 22, 2015. Model 3 excludes Governing Council meeting days on which an interest rate change is announced. In addition, Model 4 only uses repo data on monetary policy meetings after 13:45:00 CET when calculating the collateral spread. The sample period of each regression is from January 2, 2007 to June 30, 2015. Newey-West standard errors are shown in parentheses (lags are determined by $N^{\frac{1}{4}}$ (Greene, 2008), where N denotes the number of days in the sample). The superscripts *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

	Model 1		Model 2		Model 3		Model 4	
	Δ collspread 1	Δ collspread 2	Δ collspread 1	Δ collspread 2	Δ collspread 1	Δ collspread 2	Δ collspread 1	Δ collspread 2
constant	8.853 (7.546)	0.524 (0.386)	9.046 (7.566)	0.543 (0.384)	9.048 (7.564)	0.572 (0.388)	9.132 (7.563)	0.640 (0.397)
govcouncil_mp	0.640** (0.282)	0.115 (0.312)	0.630** (0.284)	0.085 (0.305)	0.567* (0.296)	-0.093 (0.291)	2.168*** (0.620)	-0.023 (0.387)
govcouncil_nonmp	-0.185 (0.373)	0.002 (0.524)	-0.216 (0.352)	-0.052 (0.481)	-0.223 (0.352)	-0.057 (0.480)	-0.249 (0.352)	-0.058 (0.480)
vstoxx _{t-1}	0.017 (0.012)	0.028 (0.017)	0.005 (0.009)	0.024 (0.017)	0.005 (0.009)	0.023 (0.017)	0.003 (0.009)	0.017 (0.017)
excessliq _{t-1}	-0.001*** (0.000)	-0.002*** (0.000)	-0.001*** (0.000)	-0.001*** (0.000)	-0.001*** (0.000)	-0.001*** (0.000)	-0.001*** (0.000)	-0.001*** (0.000)
1yearltro			9.290*** (0.146)	28.185*** (0.251)	9.284*** (0.147)	28.194*** (0.252)	9.236*** (0.148)	28.262*** (0.241)
3yearltros			6.549 (4.134)	5.725 (8.086)	6.558 (4.127)	5.722 (8.091)	6.543 (4.162)	5.674 (8.140)
zerorate			4.857*** (0.153)	4.493*** (0.150)	4.847*** (0.154)	4.484*** (0.151)	4.885*** (0.168)	4.440*** (0.153)
fullallot			16.943* (9.692)		16.943* (9.692)		17.011* (9.681)	
No. Obs	2,158	1,558	2,158	1,558	2,140	1,546	2,089	1,481
Adj. R ²	0.035	0.175	0.049	0.186	0.049	0.186	0.059	0.190

Table 17: The collateral spread of GC Pooling baskets in different time periods

This table provides descriptive statistics of the collateral spreads Eonia - GCP ECB and Eonia-GCP ECB Ext. (both in bps) in eight different time periods. The unsecured rate, Eonia, and the secured rates, GCP ECB, and GCP ECB Ext., are daily volume-weighted overnight averages. The sample period for the spread Eonia – GCP ECB is January 02, 2007 to June 30, 2015, and for the spread Eonia – GCP ECB Ext. it is November 24, 2008 to June 30, 2015. Columns 2-7 capture the number of observations, the mean, the standard error, the median, the standard deviation, the minimum and the maximum. The last two columns show the negative occurrence of the collateral spread: the second last column counts the number of negative days observed, and the last column displays the percentage of negative days of the total number of days, when trades occur in that basket.

<i>1. Jan 02, 2007 - Jul 31, 2007 (147 trading days)</i>	No. Obs.	Mean	St. Error	Median	St. Deviation	Min	Max	# neg. days	% of neg. days of obs.
Eonia – GCP ECB rate	147	-0.480	0.408	0.031	4.951	-41.537	13.085	62	42.177%
<i>2. Aug 1, 2007-Oct 08, 2008 (303 trading days)</i>									
Eonia – GCP ECB rate	303	-3.085	0.378	-2.157	6.581	-51.106	15.882	237	78.218%
<i>3. Oct 09, 2008-Jun 24, 2009 (179 trading days)</i>									
Eonia – GCP ECB rate	178	11.699	0.664	11.279	8.858	-30.045	33.279	13	7.263%
Eonia – GCP ECB Ext. rate	101	-2.822	0.936	-1.699	9.409	-25.000	16.100	58	57.426%
<i>4. Jun 25, 2009-Jul 1, 2010 (262 trading days)</i>									
Eonia – GCP ECB rate	262	5.317	0.265	5.955	4.284	-21.485	17.534	18	6.870%
Eonia – GCP ECB Ext. rate	227	2.902	0.303	3.700	4.562	-21.500	15.100	35	15.419%
<i>5. Jul 2, 2010-Dec 21, 2011 (382 trading days)</i>									
Eonia – GCP ECB rate	382	3.502	0.355	4.759	6.933	-38.457	26.991	85	22.251%
Eonia – GCP ECB Ext. rate	381	-2.284	0.413	-0.022	8.052	-45.883	13.989	191	50.131%
<i>6. Dec 22, 2011-Jun 30, 2013 (387 trading days)</i>									
Eonia – GCP ECB rate	387	7.145	0.172	7.472	3.387	-19.815	17.863	2	0.517%
Eonia – GCP ECB Ext. rate	387	5.602	0.207	6.390	4.076	-30.168	17.618	15	3.876%
<i>7. Jul 1, 2013-Jan 21, 2015 (397 trading days)</i>									
Eonia – GCP ECB rate	397	2.485	0.174	2.578	3.472	-29.940	13.737	56	14.106%
Eonia – GCP ECB Ext. rate	397	0.091	0.245	0.717	4.886	-68.919	11.043	144	36.272%
<i>8. Jan 22, 2015-Jun 30, 2015 (111 trading days)</i>									
Eonia – GCP ECB rate	111	7.321	0.255	7.429	2.687	0.912	12.887	0	0.000%
Eonia – GCP ECB Ext. rate	111	5.133	0.364	5.429	3.833	-9.723	12.270	11	9.910%